The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

GEOMETRY

Wednesday, August 20, 2025 — 12:30 to 3:30 p.m., only

MODEL RESPONSE SET

Table of Contents

25 Triangle $D'A'N'$ is the image of $\triangle DAN$ after a translation.

Explain why $\triangle D'A'N'$ must be congruent to $\triangle DAN$.

Translations, or slides, are rigid motions, so they preserve all corresponding congruencies.

Score 2: The student gave a complete and correct response.

25 Triangle $D'A'N'$ is the image of $\triangle DAN$ after a translation.

Explain why $\triangle D'A'N'$ must be congruent to $\triangle DAN$.

Translation is one of the rigid motions and the corresponding side lengths and angle measures of the triangle are preserved.

Score 2: The student gave a complete and correct response.

25 Triangle $D'A'N'$ is the image of $\triangle DAN$ after a translation.

Explain why $\triangle D'A'N'$ must be congruent to $\triangle DAN$.

A translation preserves corresponding distances, so $\triangle D'A'N' \cong \triangle DAN$ by $SSS \cong SSS$.

Score 2: The student gave a complete and correct response.

25 Triangle $D'A'N'$ is the image of $\triangle DAN$ after a translation.

Explain why $\triangle D'A'N'$ must be congruent to $\triangle DAN$.

Translations presure angle measure and distance

Score 1: The student wrote an incomplete explanation.

25 Triangle $D'A'N'$ is the image of $\triangle DAN$ after a translation.

Explain why $\triangle D'A'N'$ must be congruent to $\triangle DAN$.

A translation preserves shape and size. only a dilation changes size.

Score 1: The student wrote an incomplete explanation.

25 Triangle $D'A'N'$ is the image of $\triangle DAN$ after a translation.

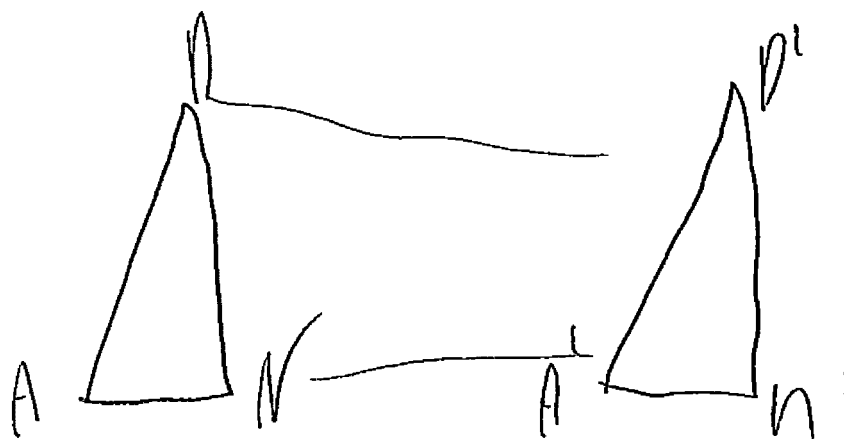Explain why $\triangle D'A'N'$ must be congruent to $\triangle DAN$.

A translation is a rigid motion and rigid motions preserve angle measure so $\triangle DAN \cong \triangle D'A'N'$

Score 1: The student wrote an incomplete explanation.

25 Triangle $D'A'N'$ is the image of $\triangle DAN$ after a translation.

Explain why $\triangle D'A'N'$ must be congruent to $\triangle DAN$.

must be ≅ because
It is a translation

Score 0: The student did not show enough relevant course-level work to receive any credit.

25 Triangle $D'A'N'$ is the image of $\triangle DAN$ after a translation.

Explain why $\triangle D'A'N'$ must be congruent to $\triangle DAN$.

D'A'N' is congruent to
DAN because they are
~~those~~ are the same shape

Score 0: The student did not show enough relevant course-level work to receive any credit.

26 The table below lists five metals and their densities.

Metal	Density (g/cm³)
Zinc	7.14
Tin	7.31
Iron	7.86
Copper	8.96
Silver	10.5

A solid metal cube has an edge length of 5 cm and a mass of 982.5 grams.
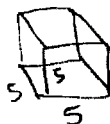
Using the table above, determine and state the type of metal from which this cube is made.

$$D = \frac{982.5}{5^3}$$

$$D = \frac{982.5}{125}$$

$$D = 7.86$$

Ans: The cube is made of Iron

Score 2: The student gave a complete and correct response.

26 The table below lists five metals and their densities.

Metal	Density (g/cm^3)
Zinc	7.14
Tin	7.31
Iron	7.86
Copper	8.96
Silver	10.5

A solid metal cube has an edge length of 5 cm and a mass of 982.5 grams.

Using the table above, determine and state the type of metal from which this cube is made.

982.5g   V = Bh

V = 25·5

V = 125 cm³

$\frac{982.5g}{125cm^3} = \boxed{7.86}$

Area of base = l·w

5·5 = 25

The solid metal cube is made of Iron. Iron weighs 7.86 g per cm³.

Score 2: The student gave a complete and correct response.

26 The table below lists five metals and their densities.

Metal	Density (g/cm^3)
Zinc	7.14
Tin	7.31
Iron	7.86
Copper	8.96
Silver	10.5

A solid metal cube has an edge length of 5 cm and a mass of 982.5 grams.

Using the table above, determine and state the type of metal from which this cube is made.

$$D = \frac{M}{V} \qquad V = b \cdot h$$

zinc

$7.14 = \frac{x}{5^3}$

892.5

tin

$7.31 = \frac{x}{5^3}$

913.75

iron

$7.86 = \frac{x}{5^3}$

982.5

copper

$8.96 = \frac{x}{5^3}$

1120

silver

$10.5 = \frac{x}{5^3}$

1312.5

The solid metal cube is made out of Iron

Score 2: The student gave a complete and correct response.

26 The table below lists five metals and their densities.

Metal	Density (g/cm³)
Zinc	7.14
Tin	7.31
Iron	7.86
Copper	8.96
Silver	10.5

A solid metal cube has an edge length of 5 cm and a mass of 982.5 grams.

Using the table above, determine and state the type of metal from which this cube is made.
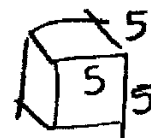
$$\frac{982.5}{5^3} = 7.86$$

Iron

Score 2: The student gave a complete and correct response.

26 The table below lists five metals and their densities.

Metal	Density (g/cm^3)
Zinc	7.14
Tin	7.31
Iron	7.86
Copper	8.96
Silver	10.5

A solid metal cube has an edge length of 5 cm and a mass of 982.5 grams.

Using the table above, determine and state the type of metal from which this cube is made.

$5 \times 5 \times 5 = 125$

$982.5 / 125 = 7.31$

tin

Score 1: The student made a computational error.

26 The table below lists five metals and their densities.

Metal	Density (g/cm^3)
Zinc	7.14
Tin	7.31
Iron	7.86
Copper	8.96
Silver	10.5

A solid metal cube has an edge length of 5 cm and a mass of 982.5 grams.

Using the table above, determine and state the type of metal from which this cube is made.

The cube is made with iron. 982.5 = 982.5 grams.

Score 1:   The student determined the correct metal but did not show work.
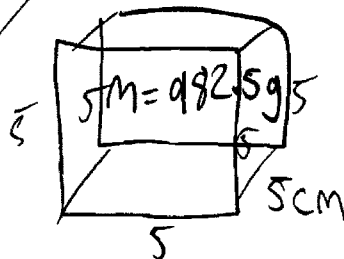
26 The table below lists five metals and their densities.

Metal	Density (g/cm³)
Zinc	7.14
Tin	7.31
Iron	7.86
Copper	8.96
Silver	10.5

Cube is like a sauer

A solid metal cube has an edge length of 5 cm and a mass of 982.5 grams.

Using the table above, determine and state the type of metal from which this cube is made.

Answer is

Iron

$M = 982.5g$

$D = \frac{982.5}{125} = 7.68$

$D = \frac{M}{V}$

$5cm$

$V = l \cdot w \cdot h$

$V = 5 \cdot 5 \cdot 5$

$V = 125$

Score 1: The student made a transcription error when finding the density of the cube, but found an appropriate metal.

26 The table below lists five metals and their densities.

Metal	Density (g/cm^3)
Zinc	7.14
Tin	7.31
Iron	7.86
Copper	8.96
Silver	10.5

A solid metal cube has an edge length of 5 cm and a mass of 982.5 grams.

Using the table above, determine and state the type of metal from which this cube is made.

982.5 grams

$7.14 = \dfrac{9825}{5cm} = 7.14$ density

Score 0: The student did not show enough correct work to receive any credit.
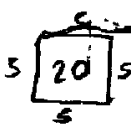
26 The table below lists five metals and their densities.

Metal	Density (g/cm^3)
Zinc	7.14
Tin	7.31
Iron	7.86
Copper	8.96
Silver	10.5

A solid metal cube has an edge length of 5 cm and a mass of 982.5 grams.

Using the table above, determine and state the type of metal from which this cube is made.

982.5

20·6 = 120

982.5/120 = 8.18

The type of metal that the cube is made out of is Cooper

Score 0: The student did not show enough correct work to receive any credit.

26 The table below lists five metals and their densities.

Metal	Density (g/cm^3)
Zinc	7.14
Tin	7.31
Iron	7.86
Copper	8.96
Silver	10.5

A solid metal cube has an edge length of 5 cm and a mass of 982.5 grams.

Using the table above, determine and state the type of metal from which this cube is made.

$$7.14$$
$$\times \quad 5$$
$$\overline{35.7}$$

$$982.5$$
$$\div \quad 5.0$$
$$\overline{196.5 \, g/cm}$$

Copper

Score 0:   The student did not show enough correct work to receive any credit.

27 The endpoints of $\overline{CAS}$ are $C(-3,1)$ and $S(7,6)$. Determine and state the coordinates of point A such that the ratio of $CA:AS$ is 3:2. 5 parts

[The use of the set of axes below is optional.]

$$\left(x_1 + \tfrac{3}{5}(\Delta x), y_1 + \tfrac{3}{5}(\Delta y)\right)$$

$$\left(-3 + \tfrac{3}{5}(10), 1 + \tfrac{3}{5}(5)\right)$$

$$(-3 + 6, 1 + 3)$$

$$A\,(3,4)$$
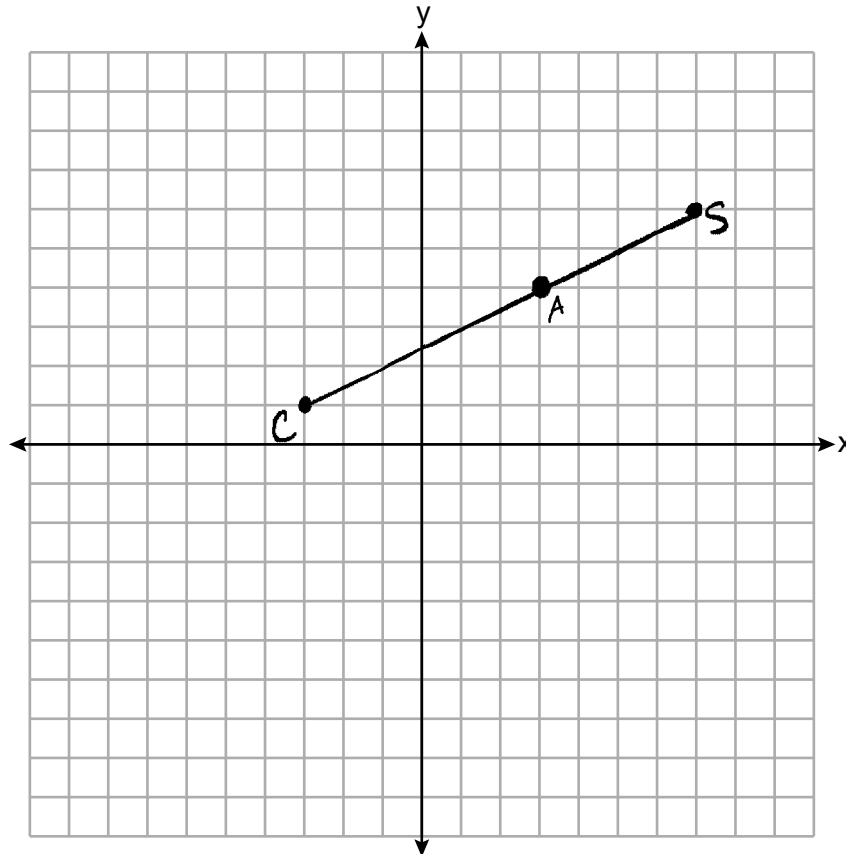
Score 2: The student gave a complete and correct response.

27 The endpoints of $\overline{CAS}$ are $C(-3,1)$ and $S(7,6)$. Determine and state the coordinates of point A such that the ratio of $CA{:}AS$ is 3:2.

[The use of the set of axes below is optional.]

point A = (3,4)
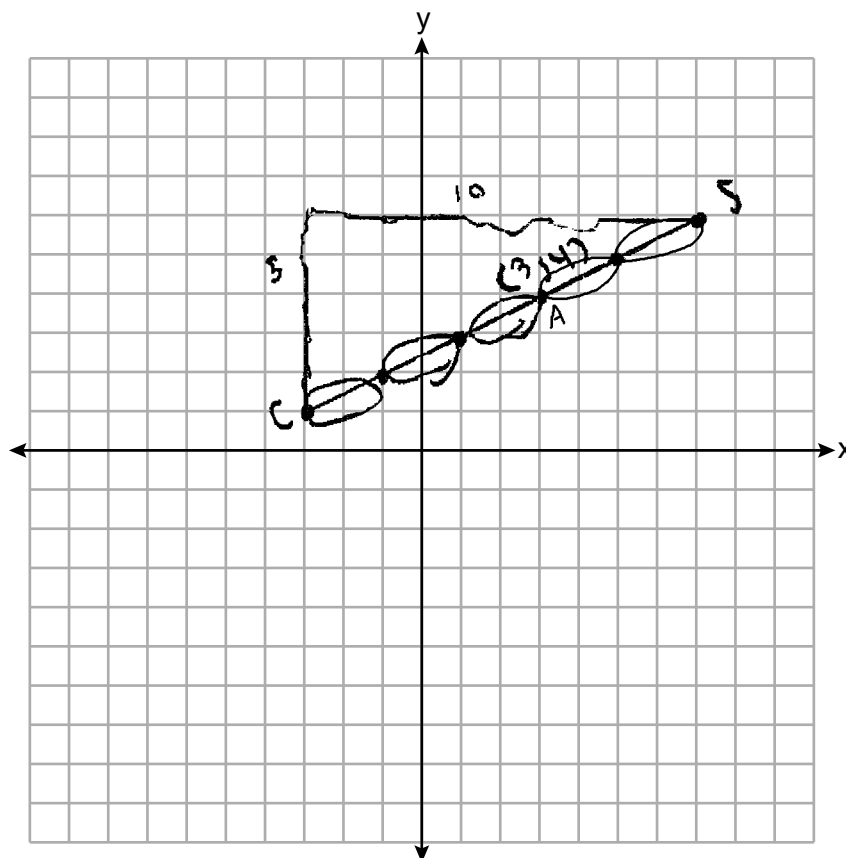
$\dfrac{5}{10} = \dfrac{1}{2}$

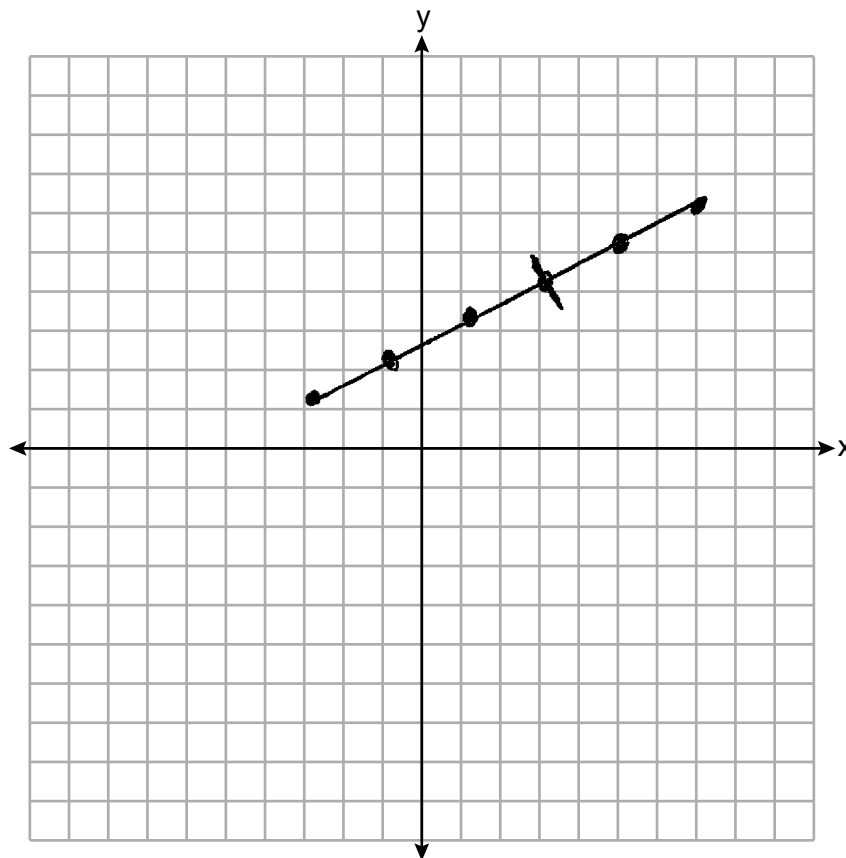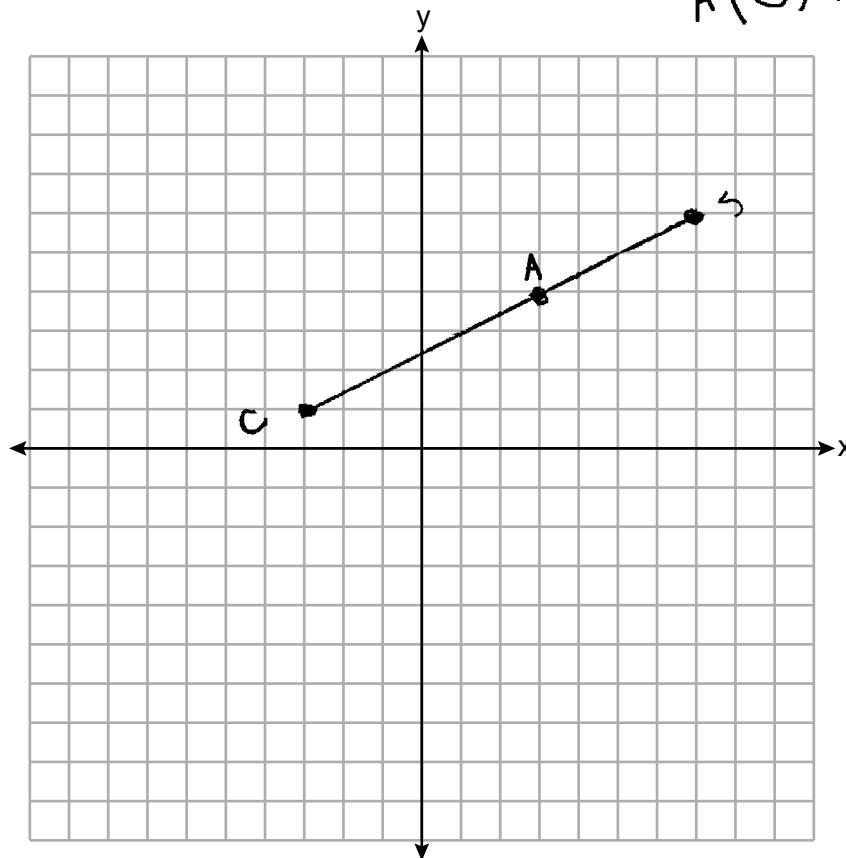**Score 2:** The student gave a complete and correct response.

27 The endpoints of $\overline{CAS}$ are $C(-3,1)$ and $S(7,6)$. Determine and state the coordinates of point A such that the ratio of $CA:AS$ is 3:2.

[The use of the set of axes below is optional.]

$A = (3, 4)$

$M = \dfrac{5}{10} = \dfrac{1}{2}$
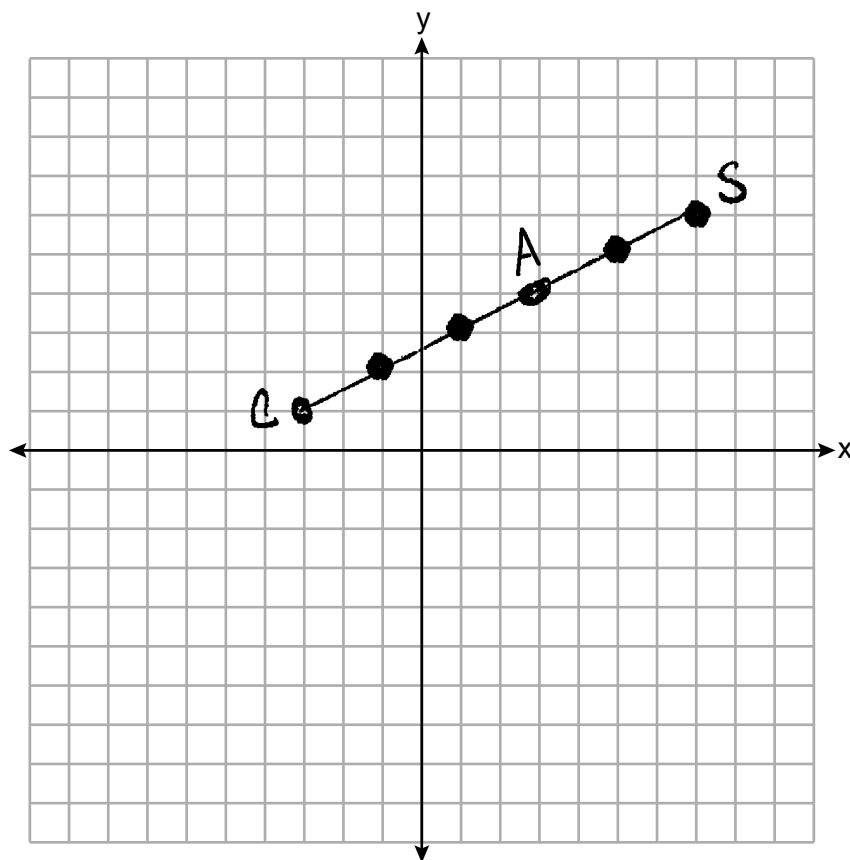
Score 2: The student gave a complete and correct response.

27 The endpoints of $\overline{CAS}$ are $C(-3,1)$ and $S(7,6)$. Determine and state the coordinates of point A such that the ratio of $CA:AS$ is 3:2.

[The use of the set of axes below is optional.]

$$\text{horz} \quad \left| \frac{3}{5} \cdot \frac{10}{1} \right| = 6$$

$$\text{Vert} = \left| \frac{3}{5} \cdot \frac{5}{1} \right| = 3$$

$$C(-3, 1)$$
$$\underline{+6 \quad +3}$$
$$A(3, 4)$$

Score 2: The student gave a complete and correct response.

27 The endpoints of $\overline{CAS}$ are $C(-3,1)$ and $S(7,6)$. Determine and state the coordinates of point A such that the ratio of $CA:AS$ is 3:2.
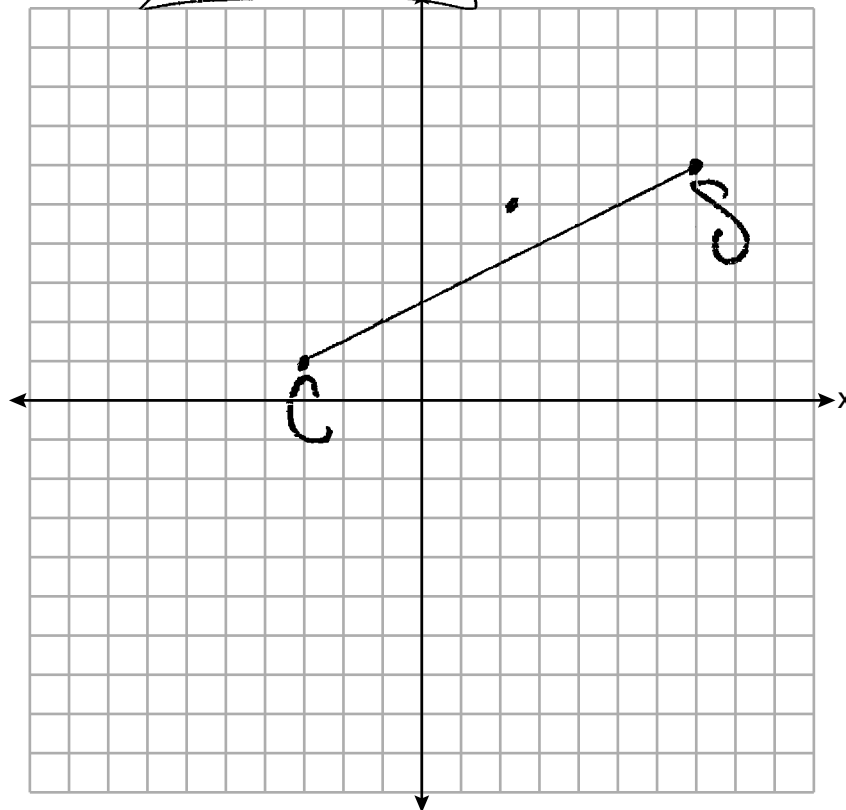
[The use of the set of axes below is optional.]

Score 1: The student correctly indicated point A, but the coordinates of A were not stated as a point.

27 The endpoints of $\overline{CAS}$ are $C(-3,1)$ and $S(7,6)$. Determine and state the coordinates of point A such that the ratio of $CA:AS$ is 3:2.
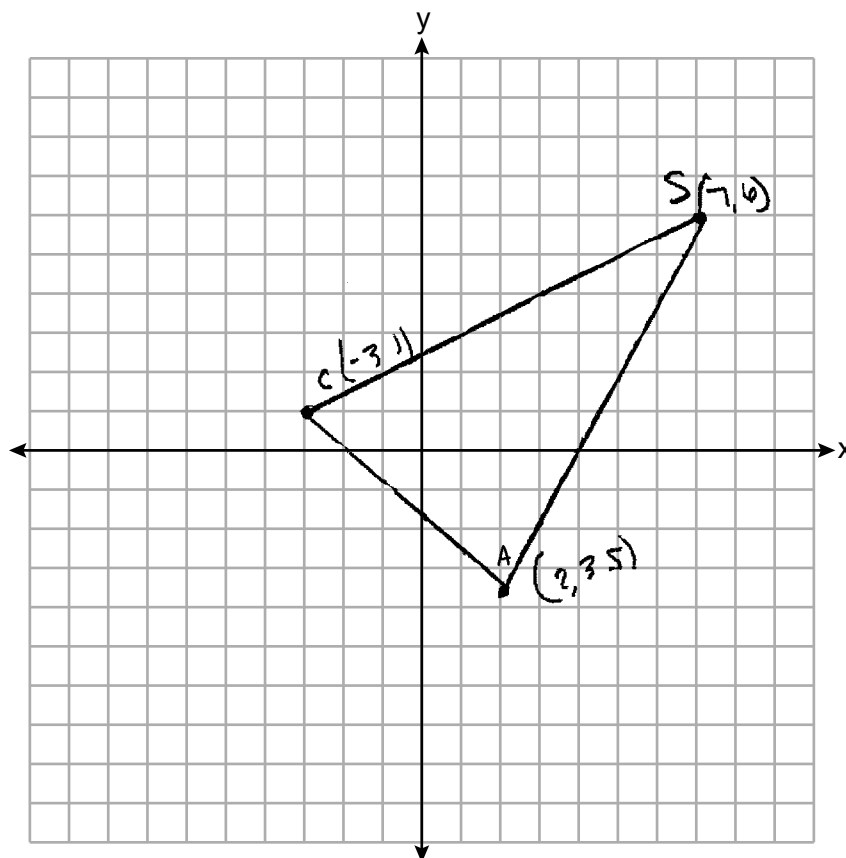
[The use of the set of axes below is optional.]

$$P = \left(\frac{a x_1 + b x_2}{a+b}, \frac{a y_1 + b y_2}{a+b} \right)$$

$$P = \left(\frac{3(-1) + 2(7)}{3+2}, \frac{3(1) + 2(6)}{3+2} \right)$$

$$P = \left(\frac{-3+14}{5}, \frac{3+12}{5} \right)$$

$$P = \left(\frac{11}{5}, 3 \right)$$

Score 0: The student used an incorrect formula and made an incorrect substitution into their formula.

27 The endpoints of $\overline{CAS}$ are $C(-3,1)$ and $S(7,6)$. Determine and state the coordinates of point A such that the ratio of $CA{:}AS$ is 3:2.

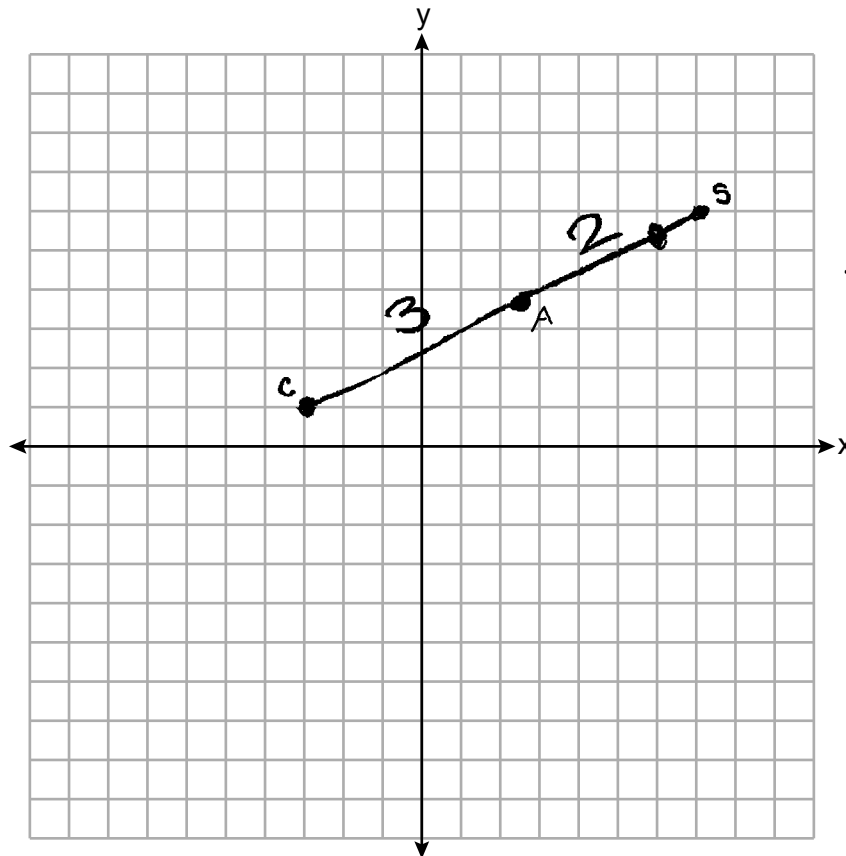[The use of the set of axes below is optional.]

$$\frac{y_1 - y_2}{x_1 - x_2}$$

$$\frac{1-6}{-3-7} = \frac{-5}{-10}$$

$$\frac{-3+7}{2} \qquad \frac{1+6}{2}$$

$$(2,\ 3.5)$$

Score 0: The student did not show enough correct relevant work to receive any credit.

27 The endpoints of $\overline{CAS}$ are $C(-3,1)$ and $S(7,6)$. Determine and state the coordinates of point A such that the ratio of $CA:AS$ is 3:2.

[The use of the set of axes below is optional.]

$$-3 + \frac{3}{5}(-3+7)$$

$$1 + \frac{3}{5}(1+6)$$

$$-.6$$

$$-5.6$$
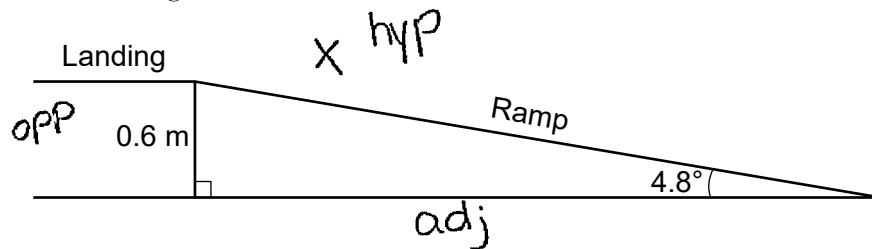
$$(-6, 5.6)$$

$\frac{3}{5}$ of the way

Score 0: The student used an incorrect formula, and made a computational error.

28 The ramp shown in the diagram below has an angle of elevation of 4.8°. The ramp is built to a landing 0.6 m above the ground.

x hyp

Landing

opp   0.6 m   Ramp

4.8°

adj

Determine and state the length of the ramp, to the *nearest tenth of a meter*.

$S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$

$$\frac{\sin(4.8)}{1} = \frac{0.6}{x}$$

$$\frac{\sin(4.8)\,x}{\sin(4.8)} = \frac{0.6}{\sin(4.8)}$$
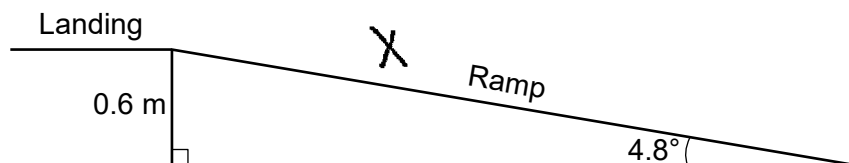
$$x = 7.17$$

$$\boxed{x = 7.2}$$

Score 2:   The student gave a complete and correct response.

28 The ramp shown in the diagram below has an angle of elevation of 4.8°. The ramp is built to a landing 0.6 m above the ground.
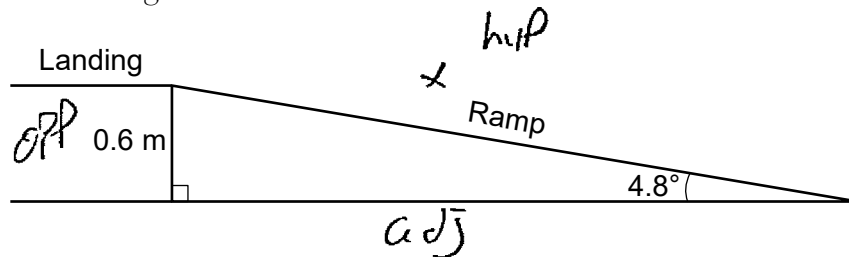
Landing

0.6 m

X

Ramp

4.8°

Determine and state the length of the ramp, to the *nearest tenth of a meter*.

SOH

$$\sin(4.8) = \frac{.6}{X}$$

$$\frac{\sin(4.8)° \cdot X}{\sin(4.8)} = \frac{.6}{\sin(4.8)}$$

$$X = 7.2 \, m$$

Score 2: The student gave a complete and correct response.

28 The ramp shown in the diagram below has an angle of elevation of 4.8°. The ramp is built to a landing 0.6 m above the ground.

Landing

hyp

∠

OPP

0.6 m

Ramp

4.8°

adj

SoH - CaH -TOa

Determine and state the length of the ramp, to the *nearest tenth of a meter*.

$$\sin(4.8) = \frac{.6}{x}$$
$$\frac{}{1}$$

$$\sin(4.8)$$
$$= .08367784 33$$
$$x6 = .50766706$$

$$ramp = \frac{1}{2} meter$$

$$ramp = .5 meters$$

Score 1: The student wrote a correct relevant trigonometric equation.

28 The ramp shown in the diagram below has an angle of elevation of 4.8°. The ramp is built to a landing 0.6 m above the ground.

Landing

0.6 m

✗ Ramp

4.8°
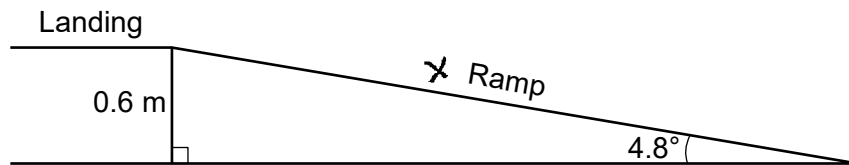
Determine and state the length of the ramp, to the *nearest tenth of a meter*.

$$x \cdot \sin(4.8) = \frac{0.6}{x} \cdot x$$

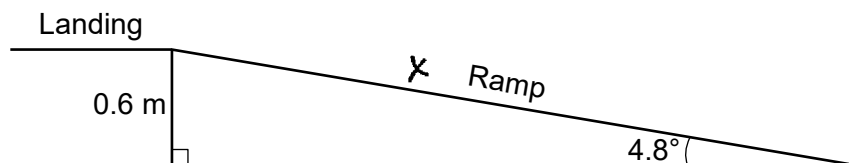$$\sin(4.8)(x) = 0.6$$

$$\frac{0.1x}{0.1} = \frac{0.6}{0.1}$$

$$x = 6$$

The length of the ramp is 6m.

Score 1: The student made a rounding error when determining sin 4.8°.

28 The ramp shown in the diagram below has an angle of elevation of 4.8°. The ramp is built to a landing 0.6 m above the ground.

Landing

0.6 m

X Ramp

4.8°

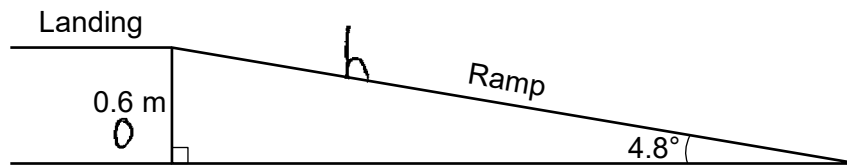Determine and state the length of the ramp, to the *nearest tenth of a meter*.

$$\tan 4.8 = \frac{0.6}{x}$$

$$\boxed{X = 7.1}$$

Score 1:   The student wrote an incorrect trigonometric equation, but solved it appropriately.

28 The ramp shown in the diagram below has an angle of elevation of 4.8°. The ramp is built to a landing 0.6 m above the ground.

Landing

0.6 m

O

Ramp

4.8°

Determine and state the length of the ramp, to the *nearest tenth of a meter*.

$$4.8^2 + 0.6^2 = C^2$$

$$23.04 + .36 = C^2$$

$$\sqrt{23.4} = \sqrt{C^2}$$

$$4.8 = C$$

Soh
Cah
Toh

$$\frac{\sin x}{4.8} = \frac{1}{0.6}$$

$$\sin(0.6) = 4.8$$

$$X = 8.5$$
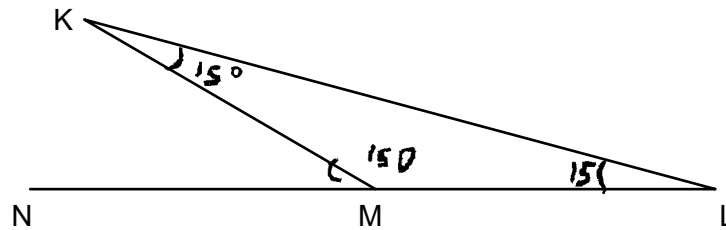
Score 0: The student did not show enough correct relevant work to receive any credit.

29 Angle *KML* is the vertex angle of isosceles triangle *KLM* below. Side $\overline{LM}$ is extended through vertex *M* to point *N*.
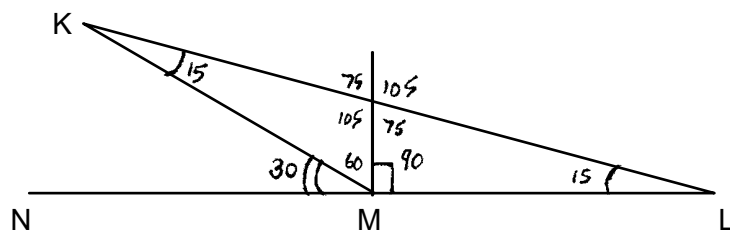
K

15°

15°

15°

N   M   L

If m∠*K* = 15°, determine and state m∠*KMN*.

isosceles triangle has the same base angle and the
sum of two interior angles = exterior angle

∠K + ∠L = ∠KMN

m∠KMN = 15 + 15

30°
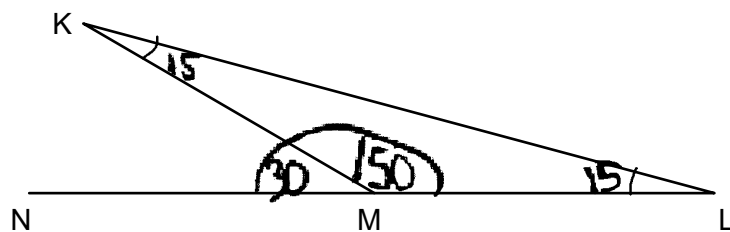
Score 2:   The student gave a complete and correct response.

29 Angle *KML* is the vertex angle of isosceles triangle *KLM* below. Side $\overline{LM}$ is extended through vertex *M* to point *N*.

If m∠*K* = 15°, determine and state m∠*KMN*.

$$m\angle KMN = 30°$$

Score 2: The student gave a complete and correct response.

29 Angle *KML* is the vertex angle of isosceles triangle *KLM* below. Side $\overline{LM}$ is extended through vertex *M* to point *N*.

If m∠*K* = 15°, determine and state m∠*KMN*.

$$m\angle KMN = 30°$$
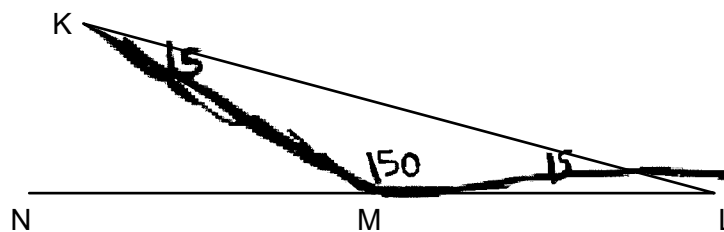
Score 2: The student gave a complete and correct response.

29 Angle *KML* is the vertex angle of isosceles triangle *KLM* below. Side $\overline{LM}$ is extended through vertex *M* to point *N*.

K

15

150

15

N   M   L

If m∠*K* = 15°, determine and state m∠*KMN*.

$$180-30 = \boxed{150}$$

Score 1:   The student determined the measure of ∠*LMK*.

29 Angle *KML* is the vertex angle of isosceles triangle *KLM* below. Side $\overline{LM}$ is extended through vertex *M* to point *N*.
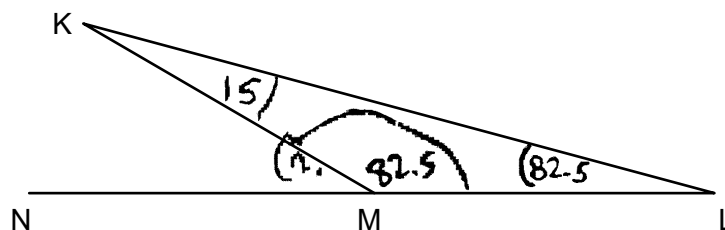
K

15

4.   82.5   82.5

N   M   L

If m∠*K* = 15°, determine and state m∠*KMN*.

$$180 - 15 = \frac{165}{2} = 82.5$$

$$180 - 82.5 = \boxed{97.5}$$

$$\boxed{m\angle KMN = 97.5}$$

Score 1:   The student made an error in determining the measure of ∠*KML*.

29 Angle *KML* is the vertex angle of isosceles triangle *KLM* below. Side $\overline{LM}$ is extended through vertex *M* to point *N*.

K ∠15

180
−15
165

165 15

N   M   L
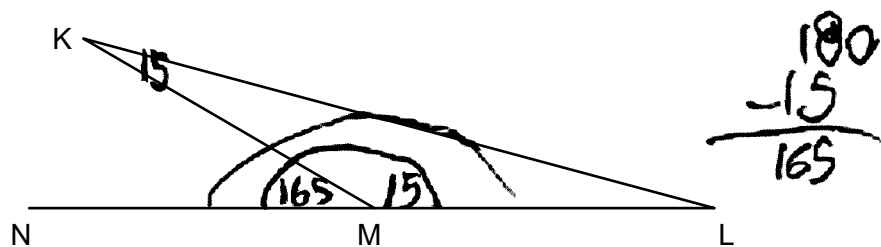
If m∠*K* = 15°, determine and state m∠*KMN* = 165

Score 1: The student made an error in determining the measure of ∠*KML*.

29 Angle *KML* is the vertex angle of isosceles triangle *KLM* below. Side $\overline{LM}$ is extended through vertex *M* to point *N*.

K

))15

180°
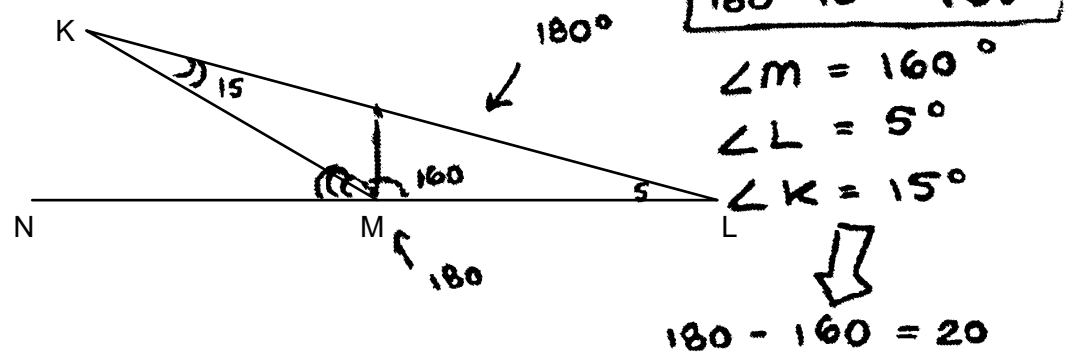
180 - 15 = 165

∠m = 160°
∠L = 5°
∠k = 15°

160

N   M   180

5   L

180 - 160 = 20

If m∠K = 15°, determine and state m∠KMN.

m∠KMN = 20°

Score 0:   The student made a conceptual error in not using isosceles triangle *KLM* and made an error in assuming m∠L = 5°.

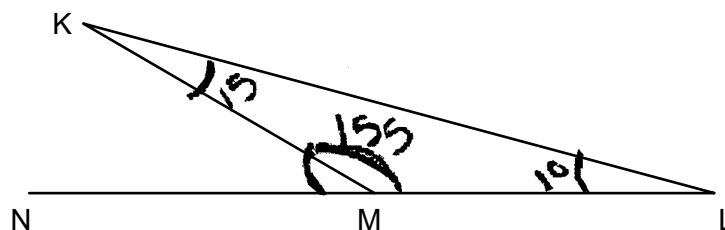**29** Angle *KML* is the vertex angle of isosceles triangle *KLM* below. Side $\overline{LM}$ is extended through vertex *M* to point *N*.

K ⟋15 ⟋155 ⟋10 N M L

If m∠*K* = 15°, determine and state m∠*KMN*.

$\triangle = 180$

$K = 15°$

$ML = 165$

$\angle = 10°$

$M = 155$

Score 0: The student did not show enough correct relevant course-level work to receive any credit.

29 Angle *KML* is the vertex angle of isosceles triangle *KLM* below. Side $\overline{LM}$ is extended through vertex *M* to point *N*.

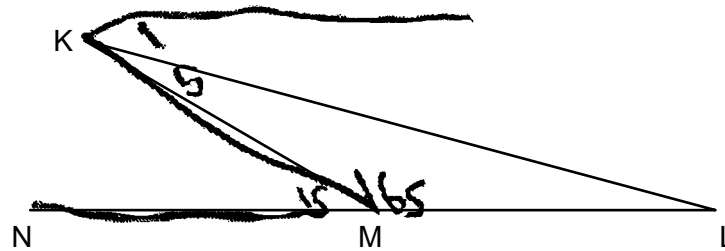If m∠*K* = 15°, determine and state m∠*KMN*.
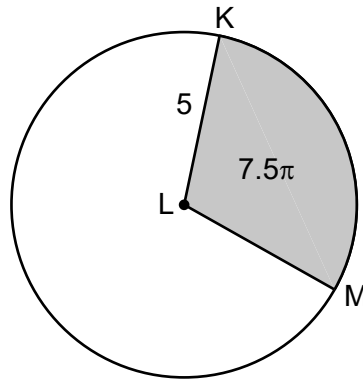
$\angle K \cong \angle NMK$

$\angle KML = 165°$

$\angle NMK = 15$

$\begin{array}{r} 180 \\ -15 \\ \hline 165 \end{array}$

Score 0: The student did not show enough correct relevant course-level work to receive any credit.

30 In the diagram below of circle *L*, the area of the shaded sector *KLM* is 7.5π and *LK* = 5.

Determine and state the degree measure of angle *KLM*, the central angle of the shaded sector.

$$x = m\angle KLM \qquad\qquad A = \pi(5)^2$$
$$\qquad\qquad\qquad\qquad = 25\pi$$

$$\frac{x}{360} = \frac{7.5\pi}{25\pi}$$

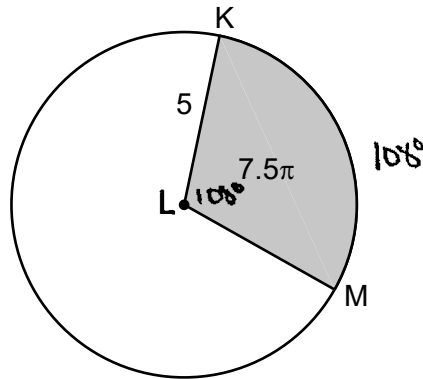$$\frac{25\pi\,x}{\pi} = \frac{2700\pi}{\pi}$$

$$\frac{25\,x}{25} = \frac{2700}{25}$$

$$\boxed{x = 108}$$

Score 2: The student gave a complete and correct response.

30 In the diagram below of circle *L*, the area of the shaded sector *KLM* is 7.5π and *LK* = 5.
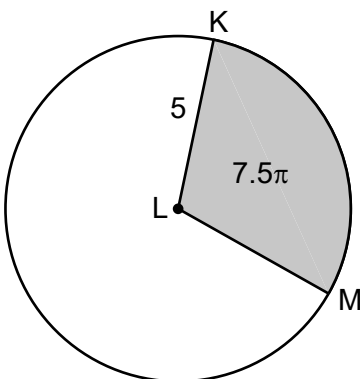

K

5

7.5π

108°

L 108°

M

Determine and state the degree measure of angle *KLM*, the central angle of the shaded sector.

$$A = \pi r^2 \left(\frac{m}{360}\right)$$

$$7.5\pi = \pi 5^2 \left(\frac{m}{360}\right)$$

$$7.5\pi = 25\pi \left(\frac{m}{360}\right)$$

$$360 \cdot 7.5\pi = \frac{25\pi m}{360} \cdot 360$$

$$\frac{2700\pi}{25\pi} = \frac{25\pi m}{25\pi}$$

$$108 = m$$

↓

$$\boxed{m\angle KLM = 108°}$$

Score 2: The student gave a complete and correct response.

30 In the diagram below of circle *L*, the area of the shaded sector *KLM* is 7.5π and *LK* = 5.

Determine and state the degree measure of angle *KLM*, the central angle of the shaded sector.

$$\frac{7.5\pi}{360} = \frac{X}{\pi(5)^2}$$

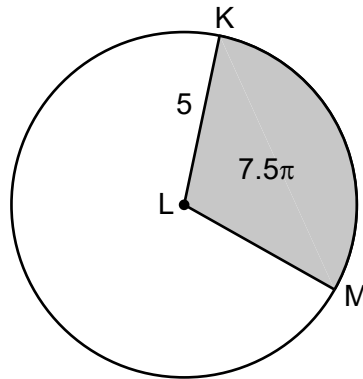$$\frac{7.5\pi}{360} = \frac{X}{25\pi}$$

$$\frac{360x}{360} = \frac{1850.5508255}{360}$$

$$X = 5.140418959$$

Score 1: The student wrote an incorrect proportion, but solved it appropriately.

30 In the diagram below of circle *L*, the area of the shaded sector *KLM* is 7.5π and *LK* = 5.

Determine and state the degree measure of angle *KLM*, the central angle of the shaded sector.

$$A = \pi r^2$$
$$A = \pi 5^2$$
$$A = 25\pi$$

$$\frac{25\pi}{7.5\pi} 3\tfrac{1}{3}\pi$$

$$\frac{7.5}{25} = \frac{1}{3}$$
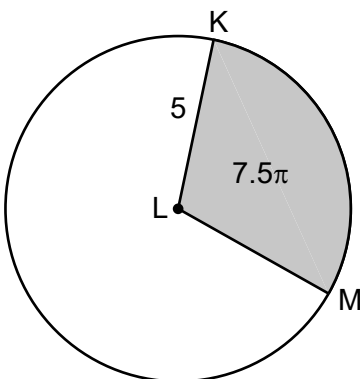
$$\frac{360}{3} = 120$$

$$\angle KLM = 120°$$

Score 1: The student made a computational error.

30 In the diagram below of circle L, the area of the shaded sector KLM is 7.5π and $LK = 5$.

Determine and state the degree measure of angle KLM, the central angle of the shaded sector.
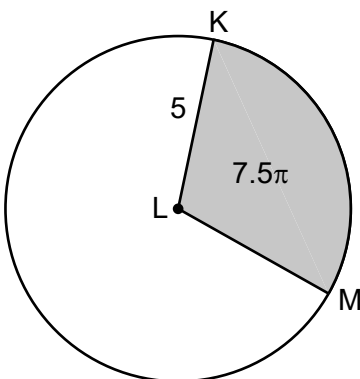
$$7.5\pi = 23.6$$
$$23.6 \times 5 = 118$$

$$\angle KLM = 118$$

Score 0: The student did not show enough correct relevant course-level work to receive any credit.

30 In the diagram below of circle L, the area of the shaded sector KLM is 7.5π and $LK = 5$.

Determine and state the degree measure of angle KLM, the central angle of the shaded sector.

$$\frac{7.5\pi}{360} =$$

$$\frac{5}{360}$$

Score 0: The student did not show enough correct relevant course-level work to receive any credit.
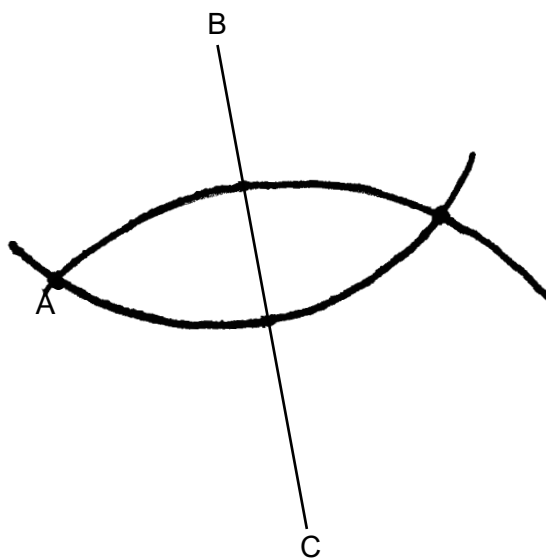
31 Using a compass and straightedge, construct the image of point A after a reflection over $\overline{BC}$.

[Leave all construction marks.]

Score 2: The student gave a complete and correct response. One arc has a radius length of $\overline{BA}$, centered at B, and the other arc has a radius length of $\overline{CA}$, centered at C. The intersection of the arcs is the image of point A after a reflection over $\overline{BC}$.

31 Using a compass and straightedge, construct the image of point A after a reflection over $\overline{BC}$.

[Leave all construction marks.]

Score 2: The student gave a complete and correct response.

31 Using a compass and straightedge, construct the image of point A after a reflection over $\overline{BC}$.

[Leave all construction marks.]

Score 2:   The student gave a complete and correct response.
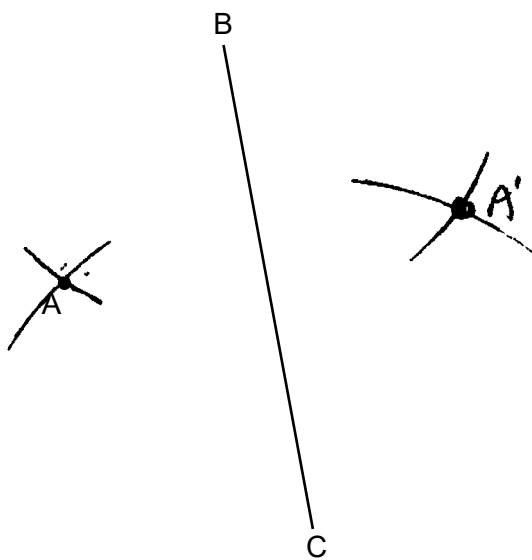
31 Using a compass and straightedge, construct the image of point A after a reflection over $\overline{BC}$.

[Leave all construction marks.]

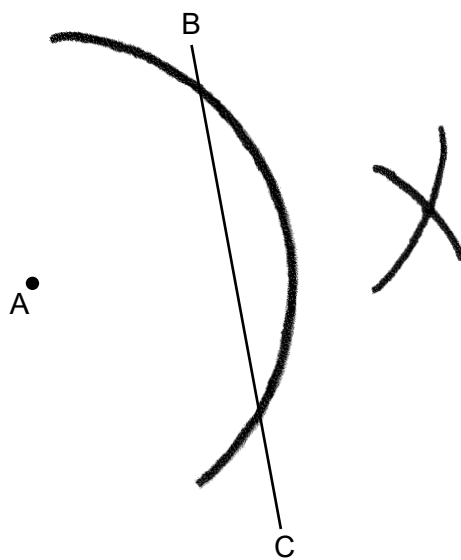**Score 2:** The student gave a complete and correct response.

31 Using a compass and straightedge, construct the image of point A after a reflection over $\overline{BC}$.

[Leave all construction marks.]

Score 1: The student made one construction error in using a radius length of $\overline{BC}$, centered at A rather than using a radius length of $\overline{AB}$, centered at B.
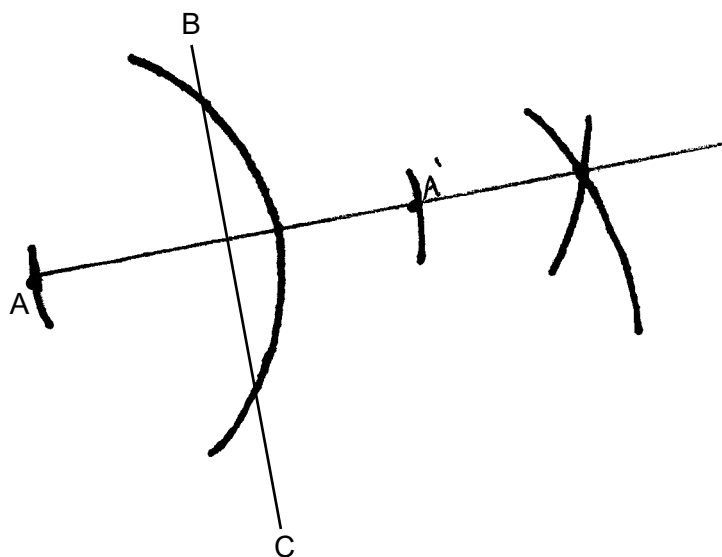
31 Using a compass and straightedge, construct the image of point *A* after a reflection over $\overline{BC}$.

[Leave all construction marks.]

Score 0: The student did not show enough correct relevant course-level work to receive any credit.
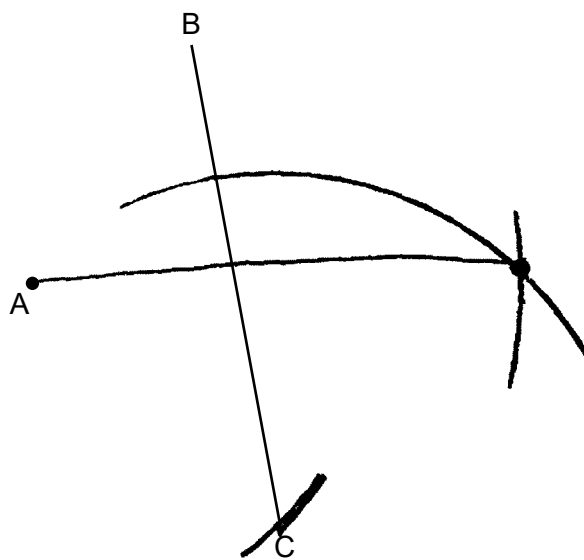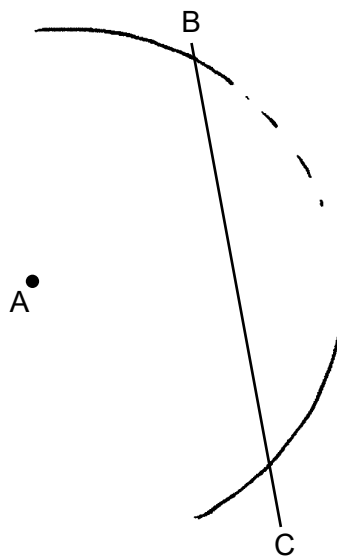
31 Using a compass and straightedge, construct the image of point *A* after a reflection over $\overline{BC}$.

[Leave all construction marks.]

B

A

A'

C

Score 0: The student did not show enough correct relevant course-level work to receive any credit.

32 Joan wants to fill an empty 75-liter fish tank with water. She uses a cylindrical bucket with a diameter of 20 cm.

Determine and state the maximum number of buckets of water, filled to an exact height of 26 cm, Joan can put into the fish tank before it overflows.

[1000 cm³ = 1 liter]

1 liter = 1000 cm³

$V = \pi r^2 h$

$V = \pi 10^2 \cdot 26$

$V = 8168.1408 \rightarrow 1$ bucket

75 liters → 75000 cm³

75000 / 8168.1408 = 9.1820

9 buckets of water

Score 4: The student gave a complete and correct response.

32 Joan wants to fill an empty 75-liter fish tank with water. She uses a cylindrical bucket with a diameter of 20 cm.

Determine and state the maximum number of buckets of water, filled to an exact height of 26 cm, Joan can put into the fish tank before it overflows.

[1000 cm^3 = 1 liter]

1 liter = 1,000 cm^3

$V = \pi r^2 h$

$= \pi (10)^2 (26)$

$= 8,168.140899$ cm^3

$\dfrac{8,168 \text{ cm}^3}{1} \cdot \dfrac{1 \text{ liter}}{1000 \text{ cm}^3} = 8.168$ liters

$\dfrac{75}{8.168} = 9.182$

$\boxed{9}$

Score 4: The student gave a complete and correct response.

32 Joan wants to fill an empty 75-liter fish tank with water. She uses a cylindrical bucket with a diameter of 20 cm.

Determine and state the maximum number of buckets of water, filled to an exact height of 26 cm, Joan can put into the fish tank before it overflows.

[1000 cm^3 = 1 liter]

$$V = \pi r^2 h$$

$$75L = 75000 \, cm^3$$

$$V = \pi 10^2 (26)$$

$$V = 8168.1409 \, cm^3$$

$$75000 \, cm^3 \div 8168.1409$$

$$\boxed{9}$$

Score 4: The student gave a complete and correct response.

32 Joan wants to fill an empty 75-liter fish tank with water. She uses a cylindrical bucket with a diameter of 20 cm.

Determine and state the maximum number of buckets of water, filled to an exact height of 26 cm, Joan can put into the fish tank before it overflows.

[1000 cm^3 = 1 liter]

$r = 10$

$V = \pi (10)^2 \cdot 26$

$100 \cdot 26 \cdot \pi$

$2600 \cdot \pi$

$V = 8168.1408$

$75 \cdot 1000$

75000

$8168.1408 \overline{)75000}$

$\boxed{9.1820}$

Buckets.

Score 3: The student made a rounding error.

32 Joan wants to fill an empty 75-liter fish tank with water. She uses a cylindrical bucket with a diameter of 20 cm.

Determine and state the maximum number of buckets of water, filled to an exact height of 26 cm, Joan can put into the fish tank before it overflows.

[1000 cm^3 = 1 liter]

$$75 \times 1000 = 75000 \text{ cm}^3 \qquad \pi 10^2(26) = 2600\pi \text{ cm}^3$$

$$\frac{75000}{2600\pi} = 90.6$$

90 buckets

Score 3:   The student made a computational error.

32 Joan wants to fill an empty 75-liter fish tank with water. She uses a cylindrical bucket with a diameter of 20 cm.

Determine and state the maximum number of buckets of water, filled to an exact height of 26 cm, Joan can put into the fish tank before it overflows.

$[1000 \text{ cm}^3 = 1 \text{ liter}]$

$$\pi (10)^2 (26)$$

$$\pi 100 (26)$$

$$8168.140899 \text{ cm}^3 / 1000$$

$$8.1681 \text{ liters}$$

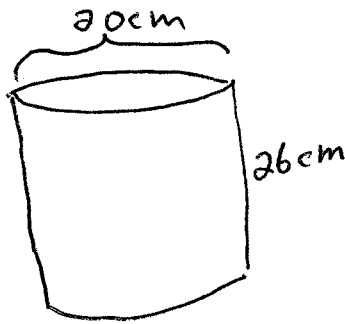$$\begin{array}{r} * \quad 75 \\ \hline 612.6 \end{array}$$

$$\boxed{612}$$

Score 3: The student correctly determined the number of liters of water in one cylindrical bucket. The student made an error in multiplying the number of liters by 75, but an appropriate number of buckets was found.

32 Joan wants to fill an empty 75-liter fish tank with water. She uses a cylindrical bucket with a diameter of 20 cm.

Determine and state the maximum number of buckets of water, filled to an exact height of 26 cm, Joan can put into the fish tank before it overflows.

[1000 cm^3 = 1 liter]

$$V = \pi r^2 h$$
$$V = \pi (10 cm)^2 (26 cm)$$
$$V = 2600\pi \ cm^3$$
$$V = 8168.140899 \ cm^3$$
$$8168.140899 \ cm^3 \cdot \frac{1 \ liter}{1000 \ cm^3} =$$
$$8.168140899 \ liters$$
$$\frac{8.168140899}{75} = .1089085453$$

1 bucket

Score 2: The student correctly determined the number of liters of water in one cylindrical bucket, but divided by 75. The student made an error when determining the number of buckets.

32 Joan wants to fill an empty 75-liter fish tank with water. She uses a cylindrical bucket with a diameter of 20 cm $\div$ 2

Determine and state the maximum number of buckets of water, filled to an exact height of 26 cm, Joan can put into the fish tank before it overflows.

[1000 cm^3 = 1 liter]

$$V = \pi r^2 h$$

$$= \pi (10)^2 (26)$$

$$= 2600\pi$$
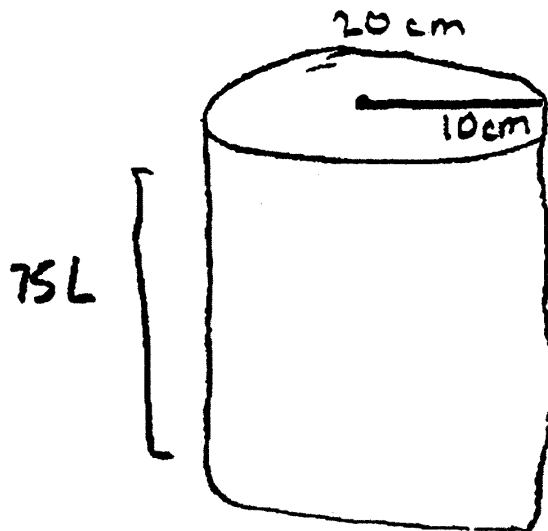
$$\frac{2600\pi}{1000} = 2.6\pi$$

$$\approx 8.168$$

Score 2: The student correctly determined the number of liters of water in the cylindrical bucket.

32 Joan wants to fill an empty 75-liter fish tank with water. She uses a cylindrical bucket with a diameter of 20 cm.

Determine and state the maximum number of buckets of water, filled to an exact height of 26 cm, Joan can put into the fish tank before it overflows.

$[1000 \text{ cm}^3 = 1 \text{ liter}]$

$$V = \pi r^2 h$$

$$= \pi 10^2 (26)$$

$$= \boxed{8,168.14 \text{ cm}^3}$$

Score 1: The student correctly determined the volume of water in one cylindrical bucket.

32 Joan wants to fill an empty 75-liter fish tank with water. She uses a cylindrical bucket with a diameter of 20 cm.

Determine and state the maximum number of buckets of water, filled to an exact height of 26 cm, Joan can put into the fish tank before it overflows.

[1000 cm^3 = 1 liter]

$$V = \pi r^2 h$$

$$V = \pi (10)^2 (26)$$

$$V = \frac{\pi 2600}{\cancel{5}}$$

108.9

109 buckets

Score 1: The student correctly determined the volume of water in one cylindrical bucket.

32 Joan wants to fill an empty 75-liter fish tank with water. She uses a cylindrical bucket with a diameter of 20 cm.

Determine and state the maximum number of buckets of water, filled to an exact height of 26 cm, Joan can put into the fish tank before it overflows.

$[1000 \text{ cm}^3 = 1 \text{ liter}]$

9

Score 1: The student determined the number of buckets, but showed no work.

32 Joan wants to fill an empty 75-liter fish tank with water. She uses a cylindrical bucket with a diameter of 20 cm.

Determine and state the maximum number of buckets of water, filled to an exact height of 26 cm, Joan can put into the fish tank before it overflows.

$[1000 \text{ cm}^3 = 1 \text{ liter}]$

$$V = \pi r^2 h$$

$$75 = \pi 10^2 (26)$$

$$V = \pi 10^2$$

Score 0: The student did not show enough correct relevant course-level work to receive any credit.

32 Joan wants to fill an empty 75-liter fish tank with water. She uses a cylindrical bucket with a diameter of 20 cm. $\frac{1}{2}\sqrt{20} = 4.4$ radius

Determine and state the maximum number of buckets of water, filled to an exact height of 26 cm, Joan can put into the fish tank before it overflows.

[1000 cm^3 = 1 liter]

$$V = \pi r^2 h$$

$$75 = \pi 10 \cancel{} h$$

$$\cancel{75 = 660}$$

$$\frac{75}{10\pi} = \frac{10 \pi h}{10\pi}$$

$$h = 2.3873240$$

$$(26)$$

$$\boxed{h = 62}$$
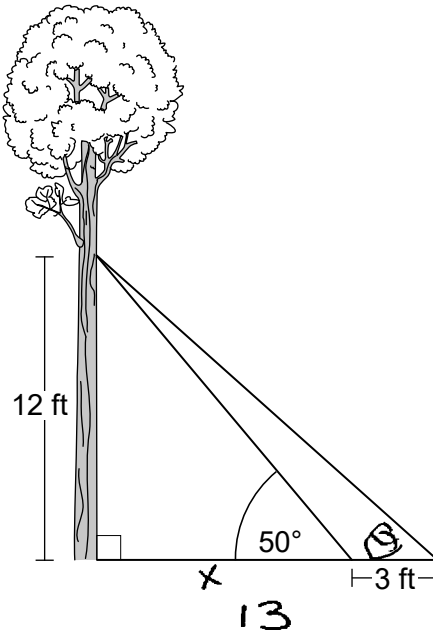
Score 0: The student did not show enough correct relevant course-level work to receive any credit.

33 As modeled in the diagram below, two cables are attached from a point on a tree 12 feet above the ground. The longer cable is anchored on the ground 3 feet farther from the tree than the shorter cable is anchored. The angle of elevation between the shorter cable and the ground is 50°.

12 ft

50°

X

13

⊢3 ft⊣

Determine and state, to the *nearest foot*, the distance from the base of the tree to the point where the longer cable is attached to the ground.

$$\tan 50 = \frac{12}{x}$$

$$\frac{\tan 50 \, x}{\tan 50} = \frac{12}{\tan 50}$$

$$10.069 + 3 = 13.069$$

$$\boxed{13 \text{ ft}}$$

Determine and state, to the *nearest degree*, the angle of elevation between the longer cable and the ground.
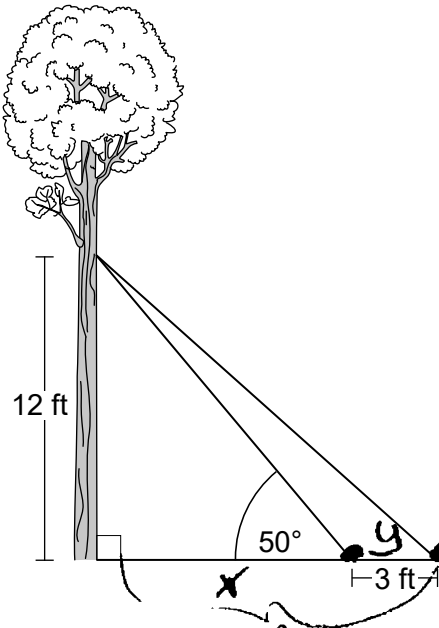
$$\tan \theta = \frac{12}{13}$$

$$\theta = 42.709$$

$$\boxed{43°}$$

Score 4: The student gave a complete and correct response.

33 As modeled in the diagram below, two cables are attached from a point on a tree 12 feet above the ground. The longer cable is anchored on the ground 3 feet farther from the tree than the shorter cable is anchored. The angle of elevation between the shorter cable and the ground is 50°.

12 ft

50°

y

3 ft

Determine and state, to the *nearest foot*, the distance from the base of the tree to the point where the longer cable is attached to the ground.

$$\tan 50 = \frac{12}{x}$$

$$x = 10.069\ldots$$

$$x \tan 50 = 12$$

$$\frac{x \tan 50}{\tan 50} = \frac{12}{\tan 50}$$

$$\begin{array}{r} 10.069 \\ +3 \\ \hline 13.069 \end{array}$$
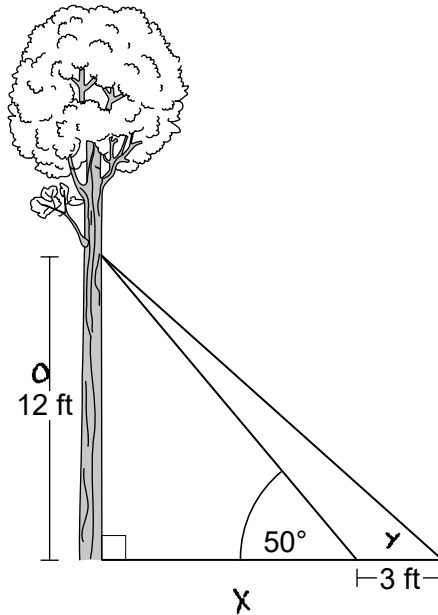
$$x = \frac{12}{\tan 50}$$

$$\boxed{13}$$

Determine and state, to the *nearest degree*, the angle of elevation between the longer cable and the ground.

$$\tan y = \frac{12}{13.069} \qquad y = \boxed{43}$$

$$\tan^{-1} \tan y = \tan^{-1} \frac{12}{13.069}$$

$$y = 42.558\ldots$$

Score 4: The student gave a complete and correct response.

33 As modeled in the diagram below, two cables are attached from a point on a tree 12 feet above the ground. The longer cable is anchored on the ground 3 feet farther from the tree than the shorter cable is anchored. The angle of elevation between the shorter cable and the ground is 50°.

Determine and state, to the *nearest foot*, the distance from the base of the tree to the point where the longer cable is attached to the ground.

$$\text{Tan } 50 = \frac{12}{X}$$

$$X(\tan 50) = 12$$
$$\frac{X(\tan 50)}{\tan 50} = \frac{12}{\tan 50}$$

$$X = 10.06919557$$
$$\underline{\quad +3 \quad}$$
$$X = 13.06919557$$
$$\boxed{X \approx 13 \text{ ft}}$$

Determine and state, to the *nearest degree*, the angle of elevation between the longer cable and the ground.

$$\text{Sin } y = \frac{12}{13}$$

$$13(\sin y) = 12$$
$$\frac{13(\sin y)}{13} = \frac{12}{13}$$

$$y \approx \text{Sin}^{-1}\left(\frac{12}{13}\right)$$

$$y \approx 67°$$

Score 3: The student used an incorrect trigonometric function to find the angle of elevation, but found an appropriate answer.

33 As modeled in the diagram below, two cables are attached from a point on a tree 12 feet above the ground. The longer cable is anchored on the ground 3 feet farther from the tree than the shorter cable is anchored. The angle of elevation between the shorter cable and the ground is 50°.

$S\frac{O}{H}$ $C\frac{A}{H}$ $T\frac{O}{A}$

12 ft

50° $y($

$x = 10$ ⊢3 ft⊣

Determine and state, to the *nearest foot*, the distance from the base of the tree to the point where the longer cable is attached to the ground.

$$(x)\ Tan(50) = \frac{12}{x}\ (x)$$

$$\frac{1.19\ x}{1.19} = \frac{12}{1.19}$$

$$x = 10.08 + 3 = \boxed{13}$$

Determine and state, to the *nearest degree*, the angle of elevation between the longer cable and the ground.
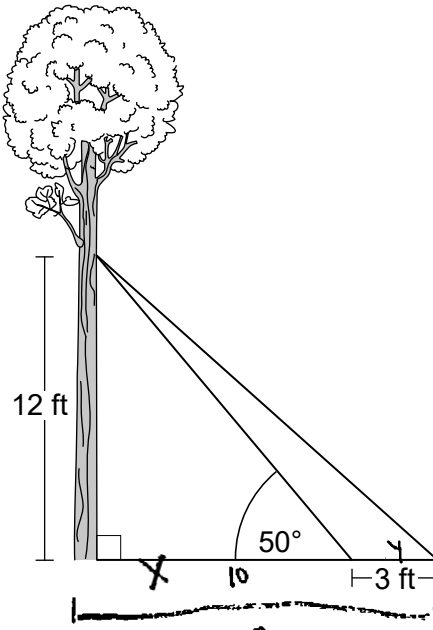
$$Tan\ y = \frac{12}{13}$$

$$y = 42.7$$

Score 3: The student made a rounding error when determining the angle of elevation.

Question 33

33 As modeled in the diagram below, two cables are attached from a point on a tree 12 feet above the ground. The longer cable is anchored on the ground 3 feet farther from the tree than the shorter cable is anchored. The angle of elevation between the shorter cable and the ground is 50°.

12 ft

50°

⊢3 ft⊣

SOH CAH TOA

Determine and state, to the *nearest foot*, the distance from the base of the tree to the point where the longer cable is attached to the ground.

$$\tan(50) = \frac{12}{x}$$

$$12 = \tan(50)x$$

$$\frac{12}{\tan(50)} = \frac{\tan(50)x}{\tan(50)}$$

$$x = 10.069 \Rightarrow x+3$$

13 ft

Determine and state, to the *nearest degree*, the angle of elevation between the longer cable and the ground.
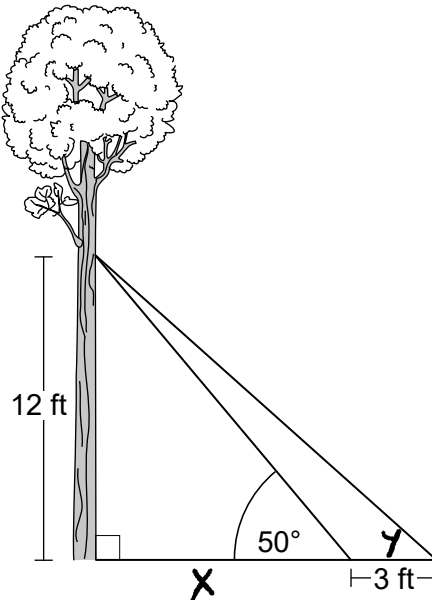
$$\tan y = \frac{12}{13}$$

45°

$$12 = \tan 13$$

$$\frac{12}{\tan(45)} = 12$$

Score 3: The student correctly determined the distance from the base of the tree to the longer cable. The student wrote a correct relevant trigonometric equation to determine the angle of elevation.

33 As modeled in the diagram below, two cables are attached from a point on a tree 12 feet above the ground. The longer cable is anchored on the ground 3 feet farther from the tree than the shorter cable is anchored. The angle of elevation between the shorter cable and the ground is 50°.

Determine and state, to the *nearest foot*, the distance from the base of the tree to the point where the longer cable is attached to the ground.

$$\text{Tan } 50 = \frac{12}{X}$$

$$X = \frac{12}{\text{Tan } 50}$$

$$\boxed{10}$$

$$\dot{X} = 10.069\ldots$$

Determine and state, to the *nearest degree*, the angle of elevation between the longer cable and the ground.

$$\text{Tan } y = \frac{12}{10}$$

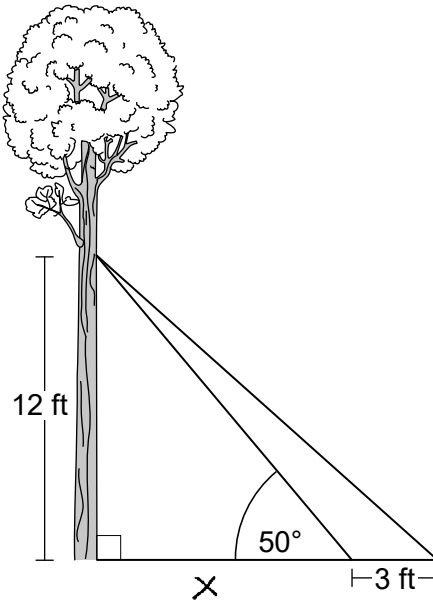$$y = \text{Tan}^{-1}\left(\frac{12}{10}\right) \qquad \boxed{50.2}$$

$$y = 50.194\ldots$$

Score 2: The student made a computational error and a rounding error.

33 As modeled in the diagram below, two cables are attached from a point on a tree 12 feet above the ground. The longer cable is anchored on the ground 3 feet farther from the tree than the shorter cable is anchored. The angle of elevation between the shorter cable and the ground is 50°.

12 ft

50°

×

⊢3 ft⊣

$$\tan 50 = \frac{12}{x}$$

$$\tan 50(x) = 12$$

Determine and state, to the *nearest foot*, the distance from the base of the tree to the point where the longer cable is attached to the ground.

Determine and state, to the *nearest degree*, the angle of elevation between the longer cable and the ground.

Score 1: The student wrote a correct relevant trigonometric equation.

33 As modeled in the diagram below, two cables are attached from a point on a tree 12 feet above the ground. The longer cable is anchored on the ground 3 feet farther from the tree than the shorter cable is anchored. The angle of elevation between the shorter cable and the ground is 50°.

12 ft

40

90   50°   13ft

⊢3 ft⊣

\
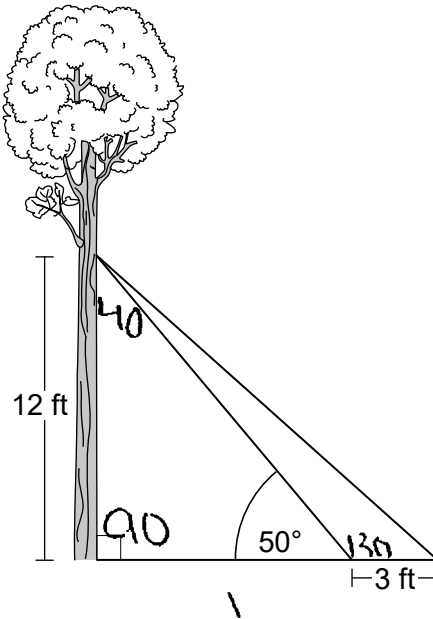
Determine and state, to the *nearest foot*, the distance from the base of the tree to the point where the longer cable is attached to the ground.

$$\approx 12 \text{ ft} \quad , \quad a^2 - \text{WM} \, 3^2 = 12^2$$

$$a^2 + 9 = 144$$

$$-9 \quad -9$$

$$\sqrt{a^2} - \sqrt{135}$$

Determine and state, to the *nearest degree*, the angle of elevation between the longer cable and the ground.

$$12 + 3 =$$

$$15 \text{ ft}$$

Score 0: The student did not show enough relevant work to receive any credit.

34 Quadrilateral *READ* has vertices with coordinates $R(-1,3)$, $E(2,7)$, $A(10,1)$, and $D(7,-3)$.

Prove *READ* is a rectangle. [The use of the set of axes below is optional.]

$$slope = \frac{y_2 - y_1}{x_2 - x_1}$$

$$distance = \sqrt{\Delta x^2 + \Delta y^2}$$

Slope of $\overline{EA} = \frac{1-7}{10-2} = \frac{-6}{8} = \boxed{-\frac{3}{4}}$

Slope of $\overline{AD} = \frac{-3-1}{7-10} = \frac{-4}{-3} = \boxed{\frac{4}{3}}$

distance of $\overline{FA}$ and $\overline{RD}$

$\sqrt{8^2 + 6^2}$ $\sqrt{8^2 + 6^2} = \sqrt{100} = \boxed{10}$

distance of $\overline{RE}$ and $\overline{AD}$

$\sqrt{3^2 + 4^2}$ $\sqrt{3^2 + 4}$

$\sqrt{25}$ $\sqrt{25}$

$\boxed{5}$ $\boxed{5}$

$\sqrt{100}$

$= \boxed{10}$

READ is a parallelogram b/c $\overline{EA} \cong \overline{RD}$ and $\overline{RE} \cong \overline{AD}$. The slope of $\overline{EA}$ $\left(-\frac{3}{4}\right)$ and $\overline{AD}$ $\left(\frac{4}{3}\right)$ are negitive recipicle so $\overline{EA} \perp \overline{AD}$ and then right ∡ A so READ is a rectangle.
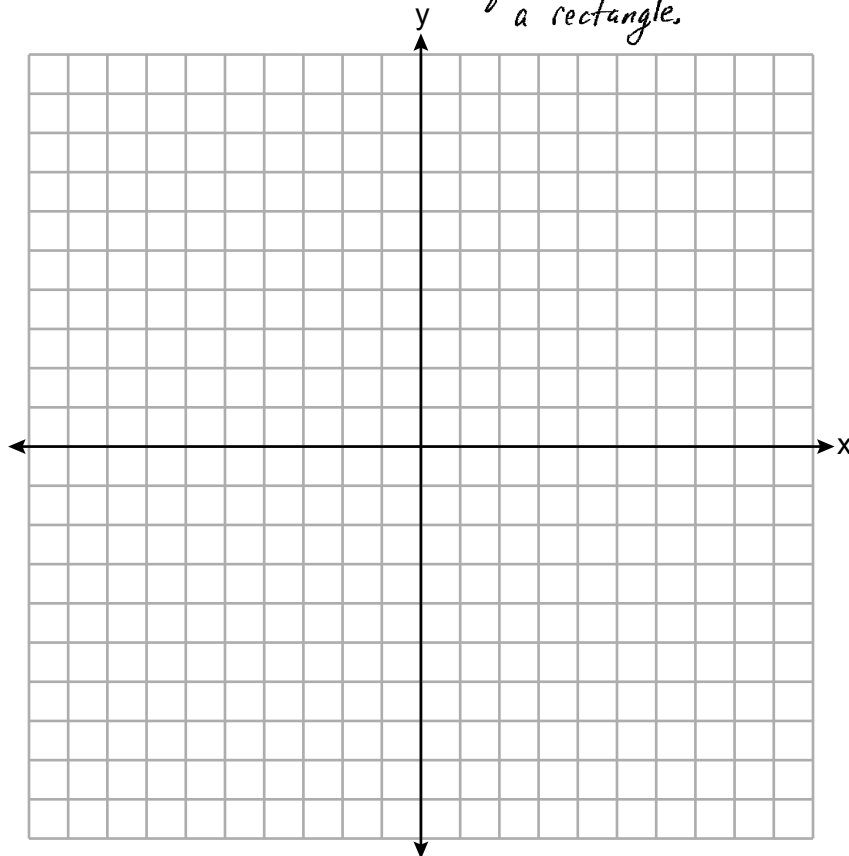
34 Quadrilateral *READ* has vertices with coordinates $R(-1,3)$, $E(2,7)$, $A(10,1)$, and $D(7,-3)$.

Prove *READ* is a rectangle. [The use of the set of axes below is optional.]

Midpt $\overline{RA} = \left(\frac{-1+10}{2}, \frac{3+1}{2}\right) = \left(\frac{9}{2}, \frac{4}{2}\right) = \left(\frac{9}{2}, 2\right)$ ⎤ same midpt

Midpt $\overline{ED} = \left(\frac{2+7}{2}, \frac{7-3}{2}\right) = \left(\frac{9}{2}, \frac{4}{2}\right) = \left(\frac{9}{2}, 2\right)$ ⎦ $\overline{RA}$ and $\overline{ED}$ bisect each other

READ is a parallelogram because the diagonals bisect each other

$d\,\overline{RA} = \sqrt{(-1-10)^2+(3-1)^2} = \sqrt{121+4} = \sqrt{125}$ ⎤ $\overline{RA} \cong \overline{ED}$

$d\,\overline{ED} = \sqrt{(2-7)^2+(7-3)^2} = \sqrt{25+100} = \sqrt{125}$ ⎦

READ is a rectangle because its a parallelogram with ≅ diagonals
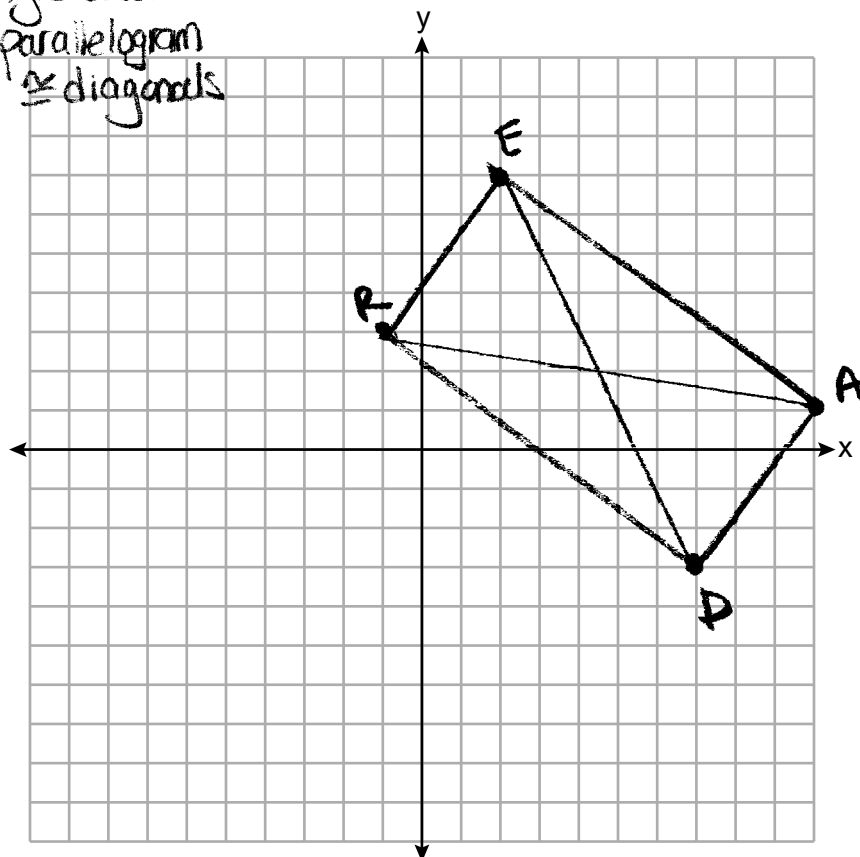
Score 4: The student gave a complete and correct response.

Question 34

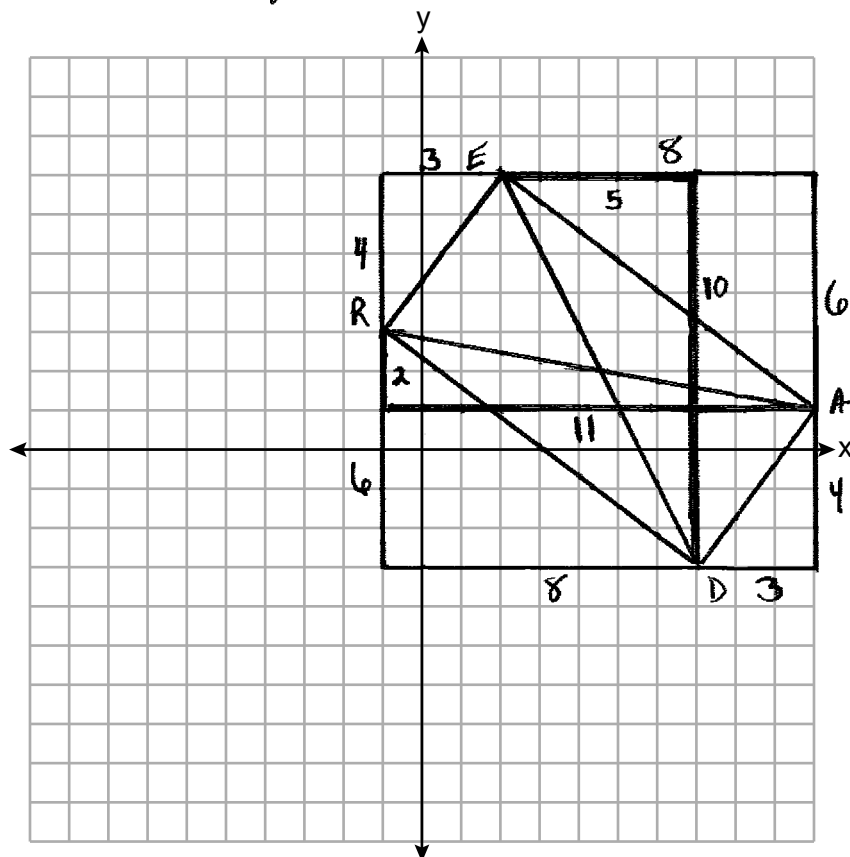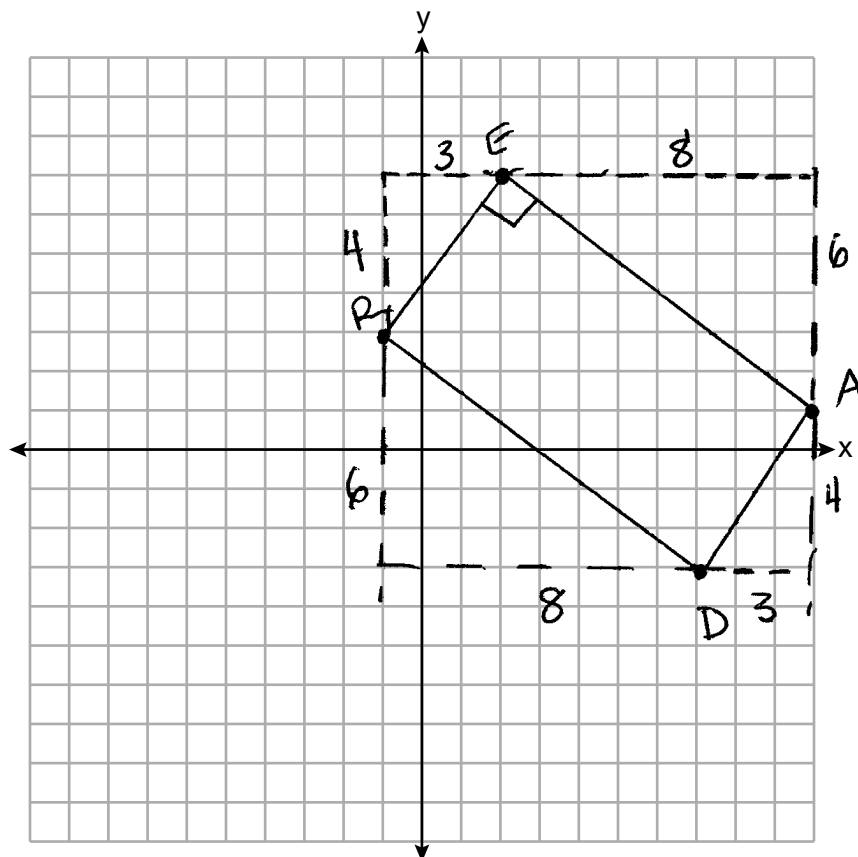**34** Quadrilateral *READ* has vertices with coordinates $R(-1,3)$, $E(2,7)$, $A(10,1)$, and $D(7,-3)$.

Prove *READ* is a rectangle. [The use of the set of axes below is optional.]

$m_{\overline{RE}} = \frac{7-3}{2-1} = \frac{4}{3}$

$\left.\right\}$ neg. reciprocals $\rightarrow \overline{RE} \perp \overline{EA} \rightarrow \angle E$ is a rt $\angle$

$m_{\overline{EA}} = \frac{1-7}{10-2} = \frac{-6}{8} = \frac{-3}{4}$

$\left.\right\}$ neg. reciprocals $\rightarrow \overline{EA} \perp \overline{AD} \rightarrow \angle A$ is a rt $\angle$

$m_{\overline{AD}} = \frac{-3-1}{7-10} = \frac{-4}{-3} = \frac{4}{3}$

$\left.\right\}$ neg. reciprocals $\rightarrow \overline{AD} \perp \overline{DR} \rightarrow \angle D$ is a rt $\angle$

$m_{\overline{DR}} = \frac{3--3}{-1-7} = \frac{6}{-8} = \frac{-3}{4}$

$\left.\right\}$ neg. reciprocal $\rightarrow \overline{DR} \perp \overline{RE} \rightarrow \angle R$ is a rt $\angle$

$m_{\overline{RE}} = \frac{4}{3}$

and all rt $\angle$'s are $\cong$

so *READ* is an equiangular quadrilateral + all equiangular quads are rectangles. $\therefore$ *READ* is a rectangle.

y

x

Score 4: The student gave a complete and correct response.

34 Quadrilateral *READ* has vertices with coordinates $R(-1,3)$, $E(2,7)$, $A(10,1)$, and $D(7,-3)$.

Prove *READ* is a rectangle. [The use of the set of axes below is optional.]

$RE = \sqrt{3^2+4^2} = \sqrt{25} = 5$
$AD = \sqrt{3^2+4^2} = \sqrt{25} = 5$ $\Big\rangle = $ so $\overline{RE} \cong \overline{AD}$

$EA = \sqrt{8^2+6^2} = \sqrt{100} = 10$
$RD = \sqrt{8^2+6^2} = \sqrt{100} = 10$ $\Big\rangle = $ so $\overline{EA} \cong \overline{RD}$

A quad w/ both pr of opp sides $\cong$ is a parallelogram so *READ* is a parallelogram.

$ED = \sqrt{5^2+10^2} = \sqrt{125}$
$RA = \sqrt{11^2+2^2} = \sqrt{125}$ $\Big\rangle = $ so $\overline{ED} \cong \overline{RA}$

A parallelogram w/ $\cong$ diagonals is a rectangle so *READ* is a rectangle.

Score 4: The student gave a complete and correct response.

34 Quadrilateral *READ* has vertices with coordinates $R(-1,3)$, $E(2,7)$, $A(10,1)$, and $D(7,-3)$.

Prove *READ* is a rectangle. [The use of the set of axes below is optional.]

$$m \overline{RE} = \frac{4}{3} \quad m \overline{EA} = -\frac{6}{8} = \frac{-3}{4}$$

$$m \overline{DA} = \frac{4}{3} \quad m \overline{RD} = -\frac{6}{8} = \frac{-3}{4}$$

$\underbrace{\qquad}_{\text{same slope} \to ||}$ $\underbrace{\qquad}_{\text{same slope.} \to ||}$

READ is a parallelogram because both pairs of opposite sides are Parallel

$$m \overline{RE} = \frac{4}{3} \quad \left. \right] \begin{array}{l} \text{opp. reciprocal slopes} \\ \overline{RE} \perp \overline{EA} \\ \angle E \text{ is a rt} \angle. \end{array}$$

$$m \overline{EA} = \frac{-3}{4}$$

Score 3: The student wrote an incomplete concluding statement when proving the rectangle.

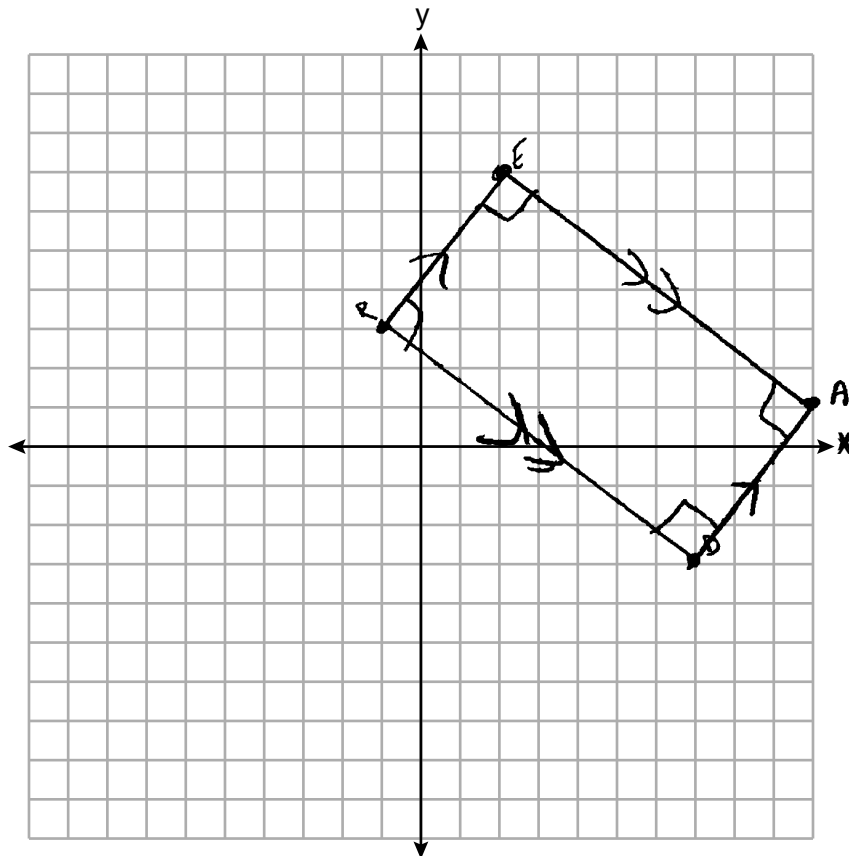**34** Quadrilateral *READ* has vertices with coordinates $R(-1,3)$, $E(2,7)$, $A(10,1)$, and $D(7,-3)$.

Prove *READ* is a rectangle. [The use of the set of axes below is optional.]

$$m_{\overline{RE}} = \frac{7-3}{2-(-1)} = \frac{4}{3}$$

$$m_{\overline{EA}} = \frac{1-7}{10-2} = \frac{-6}{8} = \frac{-3}{4}$$

$$m_{\overline{AD}} = \frac{-3-1}{7-10} = \frac{-4}{-3} = \frac{4}{3}$$

$$m_{\overline{DR}} = \frac{-3-3}{7-(-1)} = \frac{-6}{8} = \frac{-3}{4}$$

* Since the slopes of $\overline{RE}$ and $\overline{AD}$ are the same, $\overline{RE} \parallel \overline{AD}$. Since slopes of $\overline{EA}$ and $\overline{DR}$ are the same, $\overline{EA} \parallel \overline{DR}$. READ is a llogram because both pairs opp. sides are $\parallel$.

* Since slopes of $\overline{RE}$ and $\overline{EA}$ are neg. reciprocals, $\angle E$ a Right $\angle$.

* A llogram w/ a right $\angle$ is a Rectangle. $\Rightarrow$ READ is a Rectangle

y

x
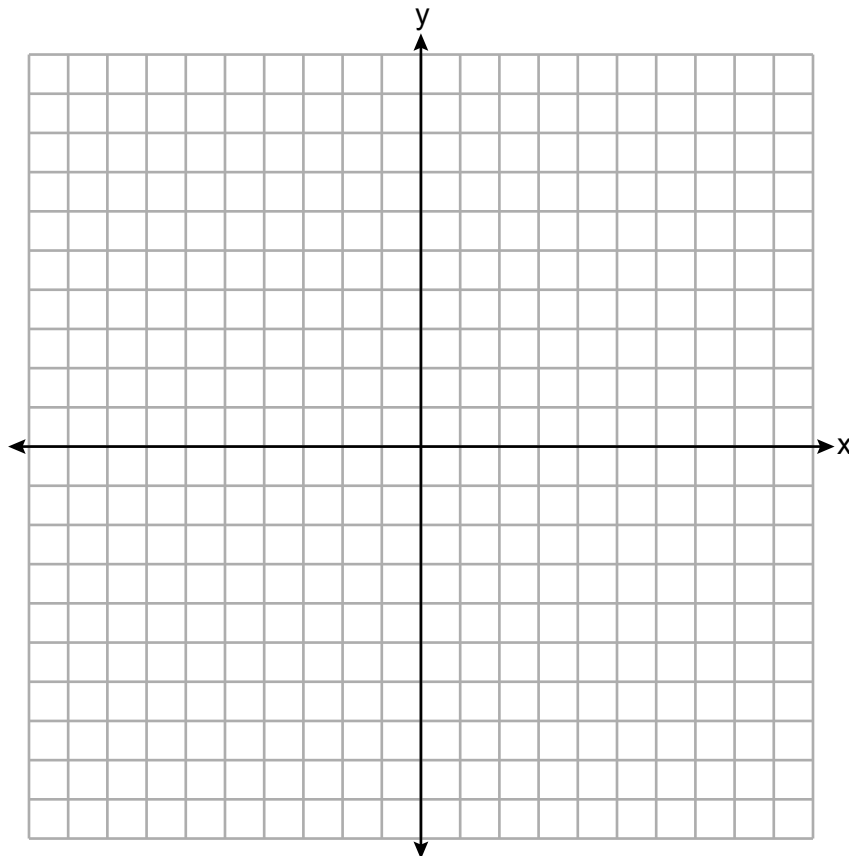
Score 3: The student wrote an incomplete conclusion in not stating $\overline{RE} \perp \overline{EA}$.

34 Quadrilateral *READ* has vertices with coordinates $R(-1,3)$, $E(2,7)$, $A(10,1)$, and $D(7,-3)$.

Prove *READ* is a rectangle. [The use of the set of axes below is optional.]

$/\!/$ $\perp$ $\angle's$

$RE = \sqrt{(2-(-1))^2+(7-3)^2} = 5$

$DA = \sqrt{(7-10)^2+(-3-1)^2} = 5$

$EA = \sqrt{(10-2)^2+(1-7)^2} = 10$

$RD = \sqrt{(7-(-1))^2+(-3-3)^2} = 10$
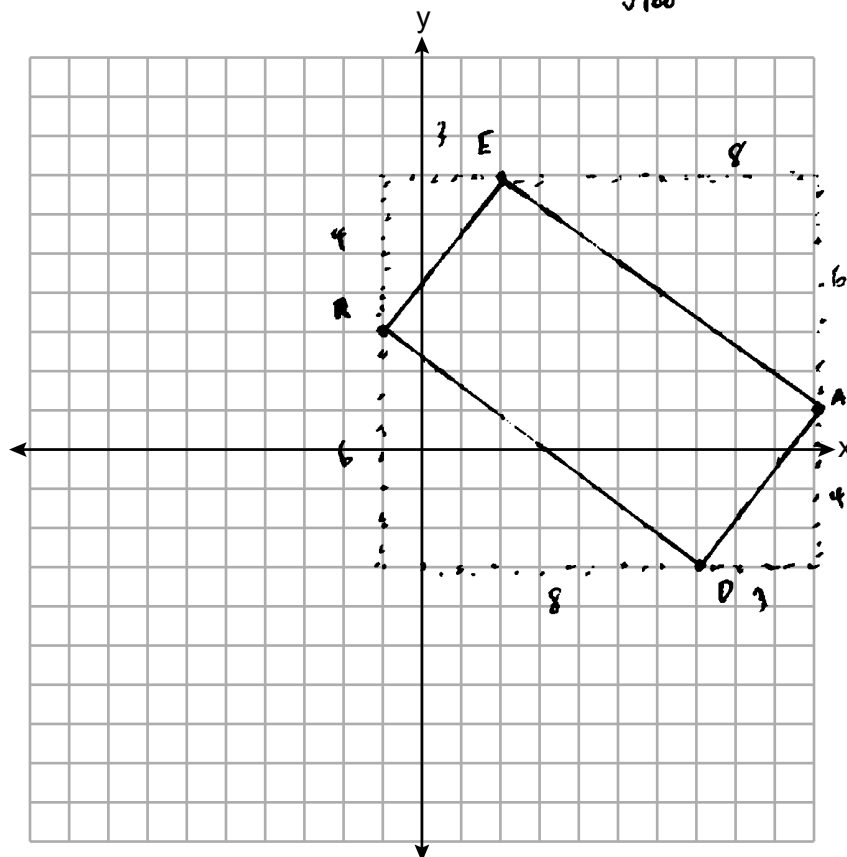
$m\overline{RE} = \frac{2-(-1)}{(7-3)} = \frac{3}{4}$

$m\overline{DA} = \frac{7-10}{-3-1} = \frac{3}{4}$

$m\overline{EA} = \frac{10-2}{1-7} = -\frac{4}{3}$

$m\overline{RD} = \frac{7-(-1)}{-3-3} = -\frac{4}{3}$

Quadrilateral READ is a parallelogram b/c $\overline{RE} \parallel \overline{DA}$ b/c equal slopes and $\overline{EA} \parallel \overline{RD}$ b/c equal slopes. $\overline{RE} \perp \overline{EA}$ $\overline{DA} \perp \overline{RD}$ b/c negative reciprocal slopes.

Score 2: The student made the same error when determining the slopes of all four sides. The student wrote an incomplete concluding statement.

34 Quadrilateral *READ* has vertices with coordinates *R*(–1,3), *E*(2,7), *A*(10,1), and *D*(7,–3).

Prove *READ* is a rectangle. [The use of the set of axes below is optional.]

$$d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)}$$

$$RE = \sqrt{(-1-2)^2 + (3-7)^2} = \sqrt{(-3)^2 + (-4)^2} = \boxed{5}$$

$$EA = \sqrt{(2-10)^2 + (7-1)^2} = \sqrt{(-8)^2 + (6)^2} = \boxed{10}$$

$$AD = \sqrt{(10-7)^2 + (1+3)^2} = \sqrt{(3)^2 + (4)^2} = \boxed{5}$$

$$DR = \sqrt{(7+1)^2 + (-3-3)^2} = \sqrt{(8)^2 + (-6)^2} = \boxed{10}$$

Using the distance formula I found that opposite
quadrilateral READ is a rectangle. I found that opposite
sides were congreunent therefore quadrilateral READ)
is a rectangle.
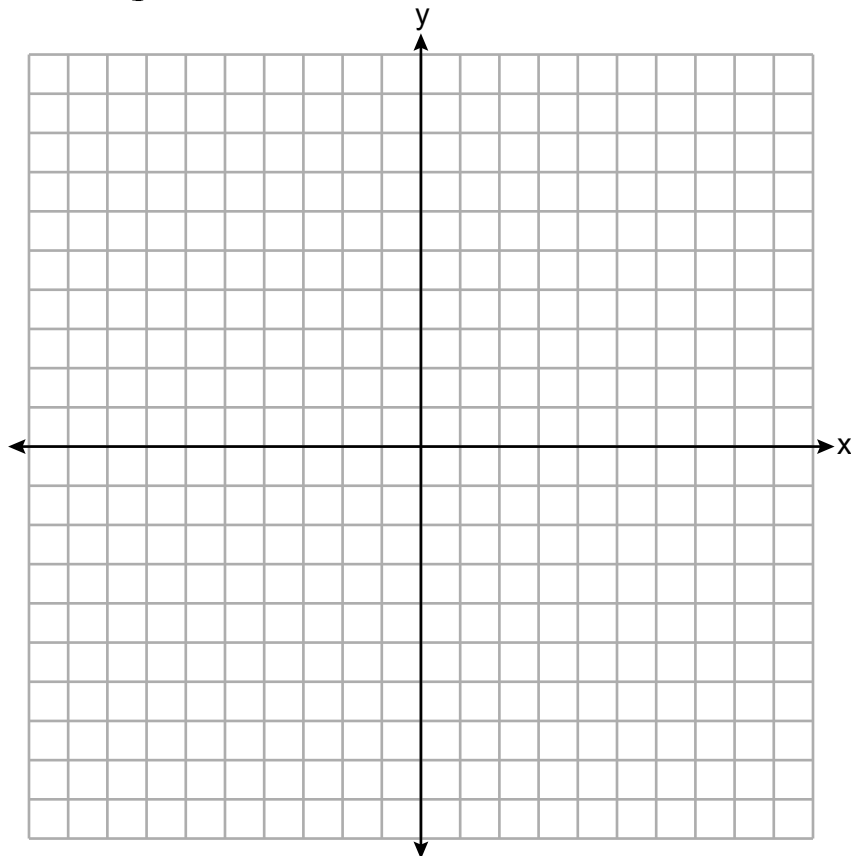
Score 2: The student made a conceptual error when proving the rectangle.

34 Quadrilateral *READ* has vertices with coordinates *R*(–1,3), *E*(2,7), *A*(10,1), and *D*(7,–3).

Prove *READ* is a rectangle. [The use of the set of axes below is optional.]

Quadrilateral READb is a rectangle because opposite sides have equal slopes and opposite sides have equal side lengths.

$$m\,\overline{RE}\ \frac{7-3}{2--1} = \frac{4}{3} \qquad m\,\overline{EA}\ \frac{7-1}{2-10} = \frac{6}{-8}$$

$$m\,\overline{AD}\ \frac{1--3}{10-7} = \frac{4}{3}$$

$$m\,\overline{RD}\ \frac{3--3}{-1-7} = \frac{6}{-8}$$

$$EA\ \sqrt{8^2+6^2} \qquad RD\ \sqrt{8^2+6^2}$$
$$\sqrt{64+36} \qquad \sqrt{64+36}$$
$$\sqrt{100} \qquad \sqrt{100}$$

$$RE\ \sqrt{4^2+3^2}$$
$$\sqrt{16+9}$$
$$\sqrt{25}$$
$$5$$

$$AD\ \sqrt{4^2+3^2}$$
$$\sqrt{16+9}$$
$$\sqrt{25}$$
$$5$$

Score 2: The student made a conceptual error when proving the rectangle.

34 Quadrilateral *READ* has vertices with coordinates *R*(–1,3), *E*(2,7), *A*(10,1), and *D*(7,–3).

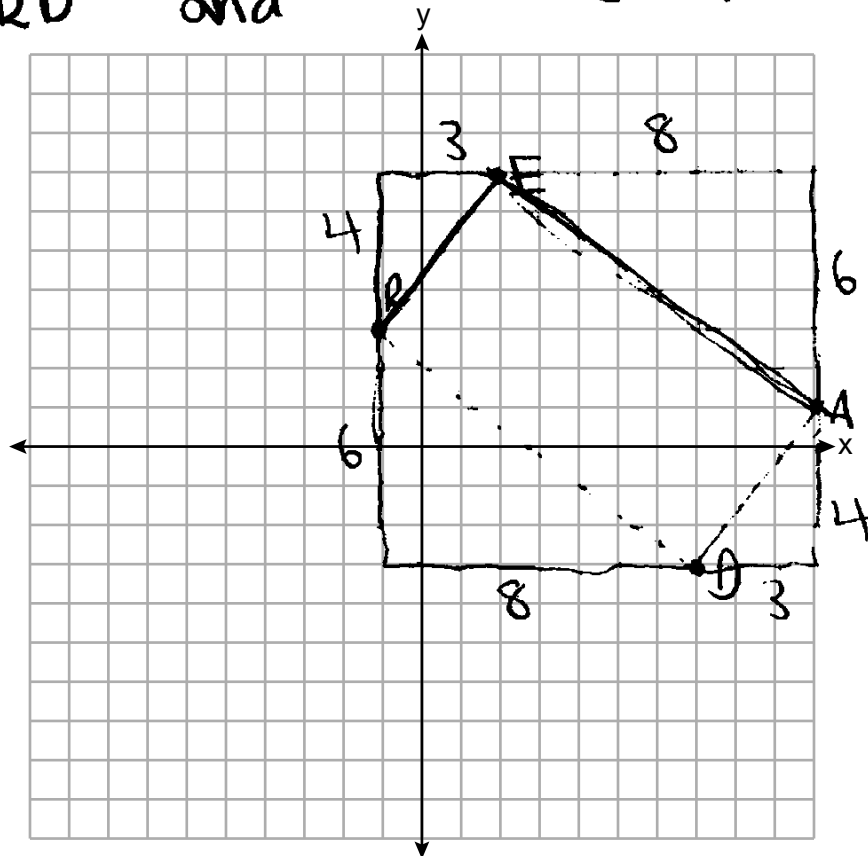Prove *READ* is a rectangle. [The use of the set of axes below is optional.]

Plan: Use slope formula to show opposite sides are parallel
Use slope formula to show adjacent sides are perpendicular:
make 90° angles, so it's a rectangle

Calc: $\overline{RE}$ $\frac{4}{3}$ $\overline{RD}$ $\frac{-6}{8}$ $\overline{DA}$ $\frac{4}{3}$ $\overline{AE}$ $\cdot\frac{6}{8}$

Conc:
$\overline{RE} \cong \overline{AD}$, $\overline{EA} \cong \overline{RD}$, so it's a rectangle
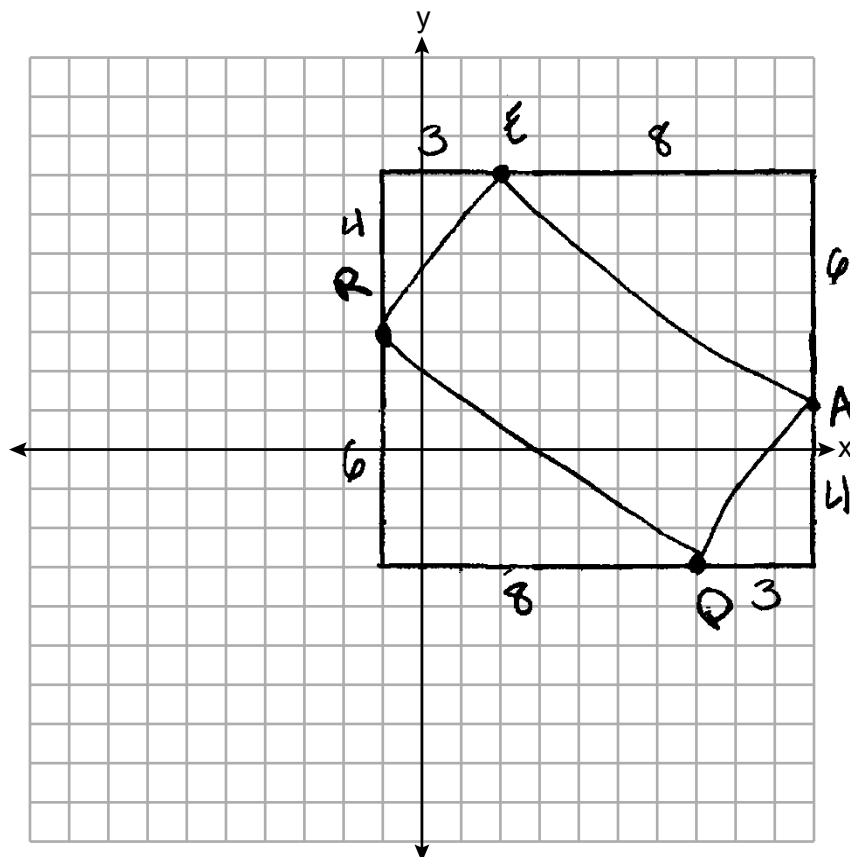
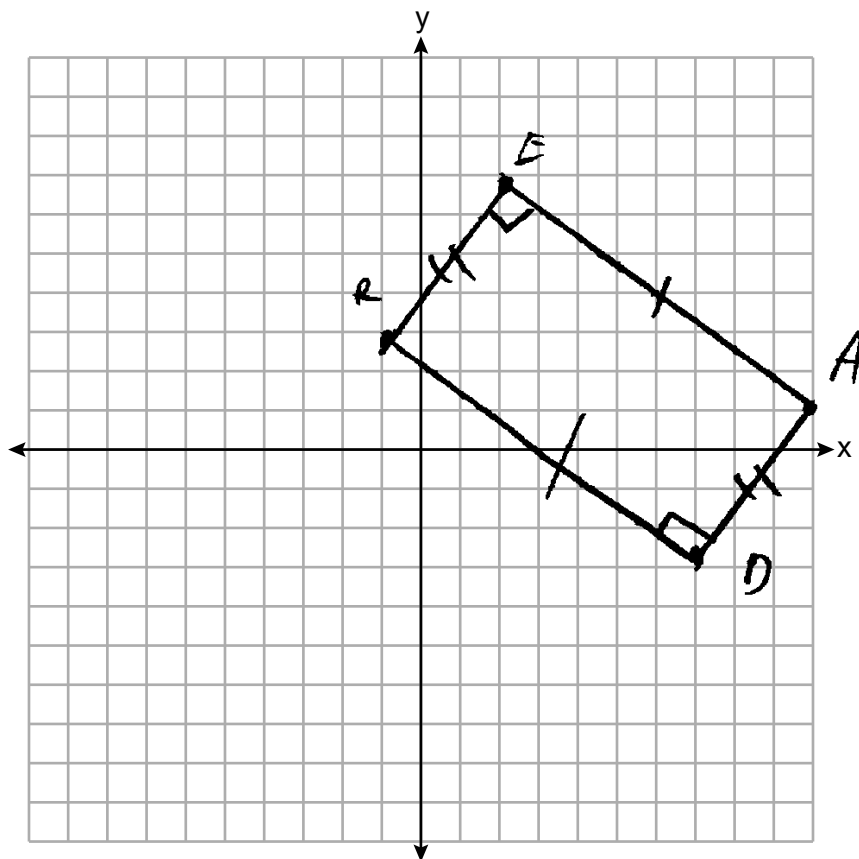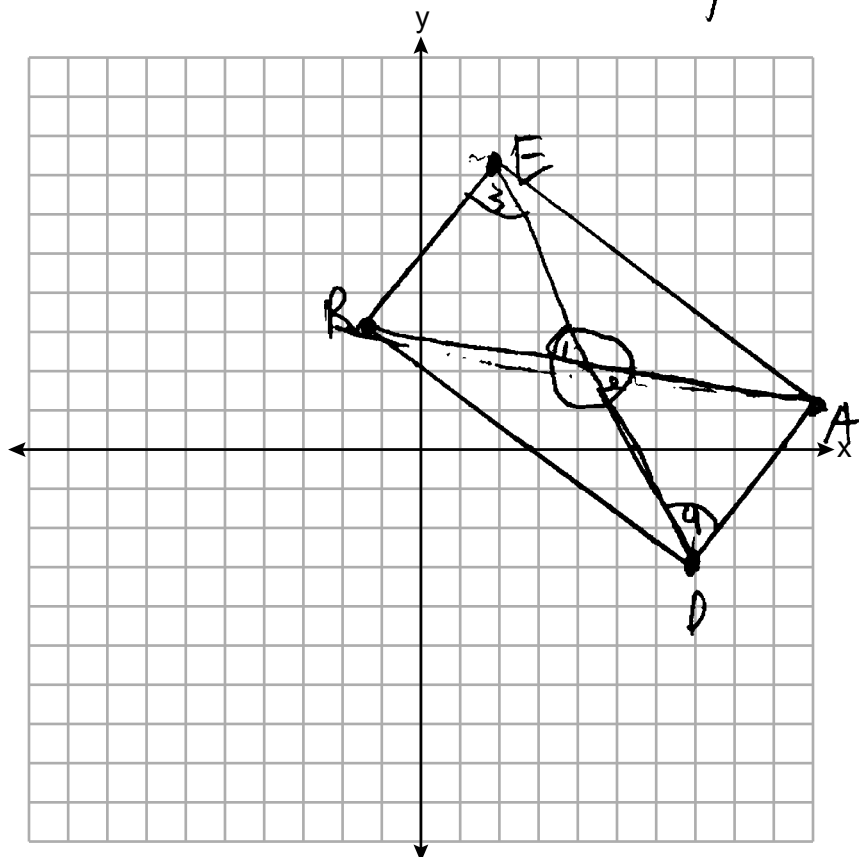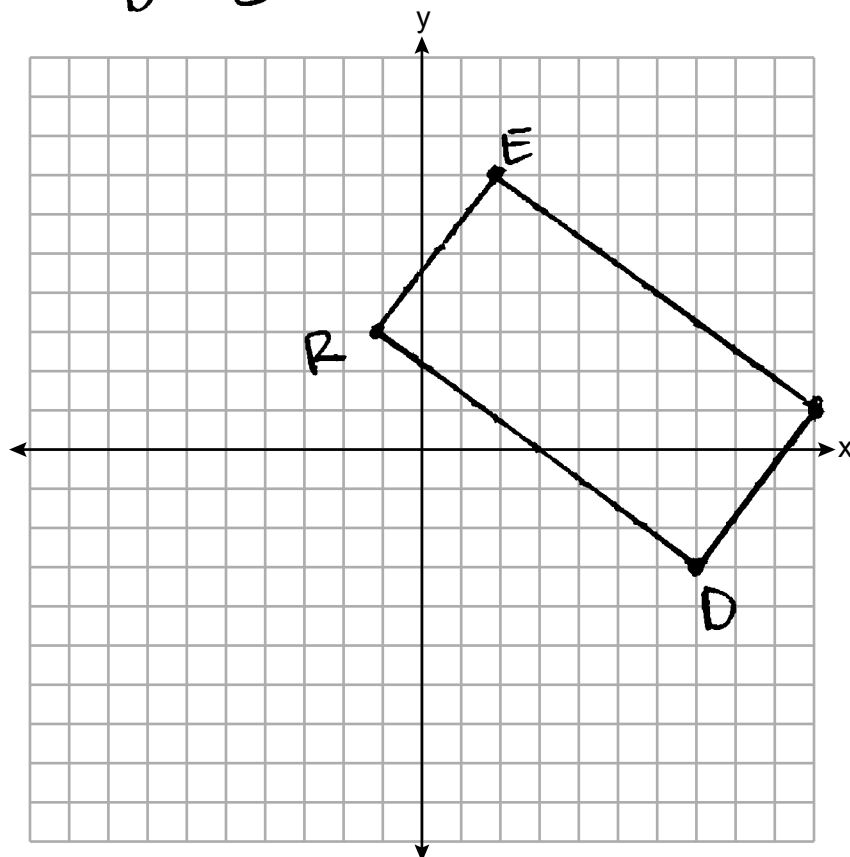**Score 1:** The student determined the four slopes of the sides of *READ*.

34 Quadrilateral *READ* has vertices with coordinates $R(-1,3)$, $E(2,7)$, $A(10,1)$, and $D(7,-3)$.

Prove *READ* is a rectangle. [The use of the set of axes below is optional.]

$$EA = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$$
$$RD = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$

$$RE = \sqrt{4^2 + 3^2} = \sqrt{25} = 5$$
$$DA = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$\overline{EA} \cong \overline{RD} \quad \text{and} \quad \overline{RE} \cong \overline{DA}$$

Score 1: The student determined the lengths of the four sides of *READ*.

34 Quadrilateral *READ* has vertices with coordinates *R*(–1,3), *E*(2,7), *A*(10,1), and *D*(7,–3).

Prove *READ* is a rectangle. [The use of the set of axes below is optional.]

Read is a rectangle

$\overline{RD}$ Slope $= \frac{6}{8}$ Because it has opposide

EA slope $= \frac{6}{8}$ congruent sides, $\overline{RD} \cong \overline{EA}$,

RE Slope $= \frac{4}{3}$ $\overline{RE} \cong \overline{AD}$,

AD Slope $= \frac{4}{3}$

Score 0:   The student did not show enough correct relevant course-level work to receive any credit.

34 Quadrilateral *READ* has vertices with coordinates *R*(–1,3), *E*(2,7), *A*(10,1), and *D*(7,–3).

Prove *READ* is a rectangle. [The use of the set of axes below is optional.]

Score 0: The student did not show enough correct relevant course-level work to receive any credit.

34 Quadrilateral *READ* has vertices with coordinates $R(-1,3)$, $E(2,7)$, $A(10,1)$, and $D(7,-3)$.

Prove *READ* is a rectangle. [The use of the set of axes below is optional.]

$$\overset{x^1 \; y^1}{(-1,3)} \overset{x^2 y^2}{(2,7)}$$

D :   RE

$$\sqrt{(x_2-x_1)^2 \, (y_2-y_1)^2}$$
$$\sqrt{(2+1)^2 + (7-3)^2}$$
$$\sqrt{9 + 16}$$
$$D = 5$$

$$\overset{x \; y}{(10,1)} \overset{x \; y}{(7-3)}$$

D : AD

$$\sqrt{(x_2-x_1)^2}$$
$$\sqrt{(7-10)^2 (-3-1)^2}$$
$$\sqrt{9+16}$$
$$D = 5$$

It has a pair of opp sides that are ≅ becaus RE and AD have the same Distance

Score 0: The student did not show enough correct relevant course-level work to receive any credit.

35 In quadrilateral *ABCD* below, side $\overline{CD}$ is extended through *D* to point *E* such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.

A   B

F

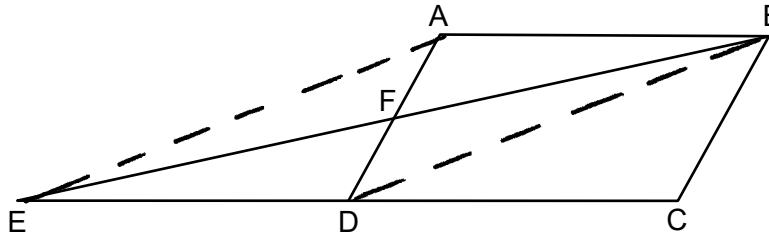E   D   C
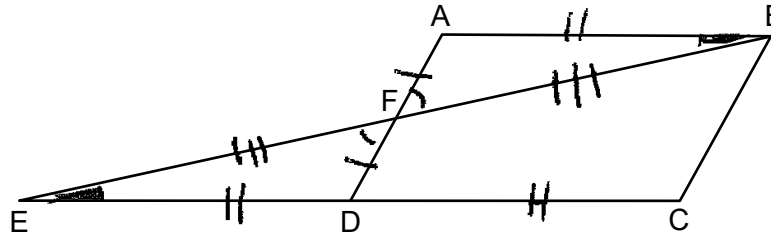
Prove *ABCD* is a parallelogram.

1. Quadrilateral AB CD, $\overline{CD}$ is extended to E $\overline{AFD}$ + $\overline{BFE}$ bisect eachother $\overline{DE} \cong \overline{DC}$	1 - Given
2. $\overline{AF} \cong \overline{FD}$ (S≅S) $\overline{BF} \cong \overline{FE}$ (S≅S)	2. Defn of a bisector
3. ∡AFB ≅ ∡DFE (a≅a)	3. Vertical ∡'s are ≅.
4. △ABF ≅ △DEF	4. SAS ≅ SAS
5. $\overline{AB} \cong \overline{DE}$	5. CPCTC
6. $\overline{AB} \cong \overline{DC}$	6. Substitution
7. ∡ ABF ≅ ∡ DEF	7. CPCTC
8. $\overline{AB} \parallel \overline{EDC}$	8. If 2 lines are cut by a transversal forming ≅ alternate interior angles, the lines are parallel
9. ABCD is a ▱	9. If one pair of opposite sides of a quadrilateral are both ≅ and parallel, it is a parallelogram.

Score 6:   The student gave a complete and correct response.

35 In quadrilateral *ABCD* below, side $\overline{CD}$ is extended through *D* to point *E* such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.

A   B

F

E   D   C

Prove *ABCD* is a parallelogram.

S	R
1. quad ABCD, $\overline{CD}$ extended thru D to E so $\overline{AFD}$, $\overline{BFE}$ bisect each other, $\overline{DE} \cong \overline{DC}$	1. given
2. Draw auxiliary lines $\overline{EA}$ and $\overline{DB}$.	2. Between any 2 points there exists exactly one line segment.
3. EABD is a parallelogram	3. If the diagonals of a quad bisect each other, it is a parallelogram.
4. $\overline{AB} \parallel \overline{DE}$, so $\overline{AB} \parallel \overline{DC}$	4. Oppo. sides of a p-gram are $\parallel$.
5. $\overline{AB} \cong \overline{DE}$	5. Oppo. sides of a p-gram are $\cong$.
6. $\overline{AB} \cong \overline{DC}$	6. Substitution
7. ABCD is a parallelogram	7. If a quad. has one set of opposite sides $\cong$ and $\parallel$, then it is a parallelogram.

Score 6:   The student gave a complete and correct response.

35 In quadrilateral *ABCD* below, side $\overline{CD}$ is extended through *D* to point *E* such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.
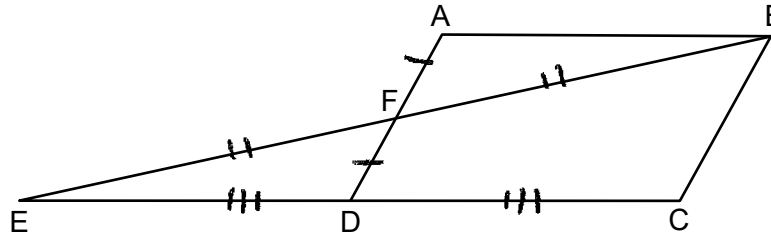
Prove *ABCD* is a parallelogram.

Given quad. ABCD, $\overline{CD}$ extended through D to E so $\overline{AFD}$ & $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.

A bisector divides a segment at its midpoint, so F is the midpoint of $\overline{EB}$.

As $\overline{DE} \cong \overline{DC}$, D is the midpoint of $\overline{EC}$.

So $\overline{DF}$ is the midsegment of $\triangle ECB$.

A midsegment is parallel to the third side of a $\triangle$, so $\overline{DF} \parallel \overline{BC}$, and so $\overline{DA} \parallel \overline{BC}$.

Since $\overline{EFB}$ bisects $\overline{AFD}$, $\overline{DF} \cong \overline{AF}$, so 2DF = DA.

A midsegment is half the length of the third side of a $\triangle$, so $2 \cdot DF = BC$. As $2 \cdot DF = DA$, then $\overline{DA} \cong \overline{BC}$ by substitution.

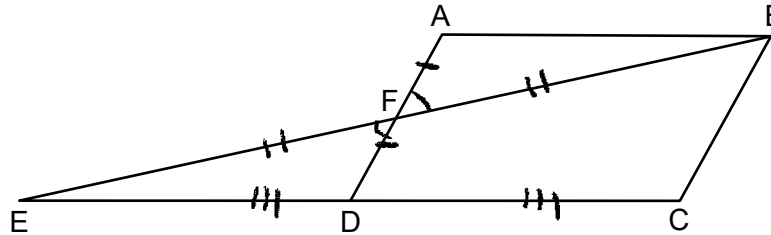Quad. ABCD has one pair of sides ($\overline{DA}$ and $\overline{BC}$)

that are both parallel

and congruent, so ABCD is a parallelogram.

Score 6: The student gave a complete and correct response.

35 In quadrilateral *ABCD* below, side $\overline{CD}$ is extended through *D* to point *E* such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.

Prove *ABCD* is a parallelogram.

Score 6: The student gave a complete and correct response.

35 In quadrilateral *ABCD* below, side $\overline{CD}$ is extended through *D* to point *E* such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.
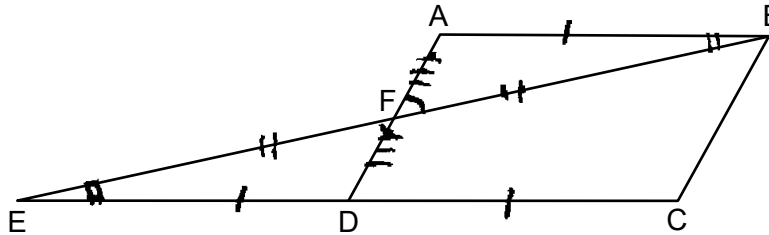

Prove *ABCD* is a parallelogram.

Statement	Reason
Quad $ABCD$, 1) $\overline{AFD}$ & $\overline{BFE}$ bisect each other, $\overline{DE} \cong \overline{DC}$ 2) $\overline{AF} \cong \overline{DF}$, $\overline{FF} \cong \overline{BF}$	1) Given 2) when a segment is bisected, it makes 2 ≅ segments
3) ∡AFB ≅ ∡DFE	3) vertical ∡s are ≅
4) △DFE ≅ △AFB	4) SAS
5) $\overline{AB} \cong \overline{DE}$ ∡FED ≅ ∡FBA	5) CPCTC
6) $\overline{AB} \parallel \overline{EDC}$	6) if alt. int. ∡s are ≅ the lines are ∥
7) ∥ogram ABCD	7) one pair opp sides ∥ & ≅ → parallogram

Score 5: The student had one statement and reason missing to prove step 7.

35 In quadrilateral *ABCD* below, side $\overline{CD}$ is extended through *D* to point *E* such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.
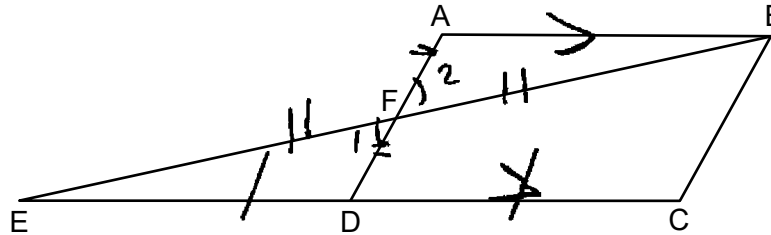

Prove *ABCD* is a parallelogram.

S	R
① $\overline{CD}$ is extended to point E, $\overline{AFD}$ and $\overline{BFE}$ bisect each other and $\overline{DE}\cong\overline{DC}$ Quad ABCD	① Given
② $\overline{AF} \cong \overline{FD}$ $\overline{EF} \cong \overline{FB}$	② Defination of a segment bisector
③ $\angle AFB \cong \angle DFE$	③ Vertical angles are congruent.
④ $\triangle EDF \cong \triangle BAF$	④ SAS
⑤ $\overline{AB} \cong \overline{DE}$ $\angle E \cong \angle FBA$	⑤ CPCTC
⑥ $\overline{AB} \cong \overline{DC}$	⑥ Subtitution
⑦ $\overline{AB} \parallel \overline{EC}$	⑦ If 2 alternate interior angle are congruent, then the lines are parallel.
⑧ ABCD is parallelogram	⑧ If line is congruent and parallel, then it is parablelogram.

Score 5: The student had an incorrect reason in step 8.

35 In quadrilateral *ABCD* below, side $\overline{CD}$ is extended through *D* to point *E* such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.
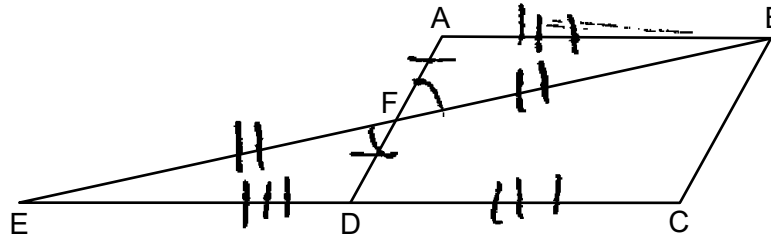

Prove *ABCD* is a parallelogram.

S	R
1) In quad ABCD below, side $\overline{CD}$ is extended to point E such that AFD and BFE bisect each other, $\overline{DE} \cong \overline{DC}$	1) Given
2) $\angle 1 \cong \angle 2$	2) Vertical ∠'s are ≅
3) F is the md pt of $\overline{AFD}$ F is the md pt of $\overline{BFE}$	3) Segment bisectors intersect a segment at its md pt
4) $\overline{AF} \cong \overline{DF}$ $\overline{EF} \cong \overline{BF}$	4) md pts ÷ a segment into 2 ≅ segments
5) $\triangle EFD \cong \triangle BFA$	5) SAS
6) $\overline{DE} \cong \overline{AB}$	6) Corresponding parts of ≅ △'s are ≅
7) $\overline{AB} \cong \overline{DC}$	7) Substitution
8) Quad ABCD is a ▱	8) In a quad, if one pair of sides is ≅, it is a ▱

Score 4: The student made a conceptual error in step 8.

35 In quadrilateral *ABCD* below, side $\overline{CD}$ is extended through *D* to point *E* such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.
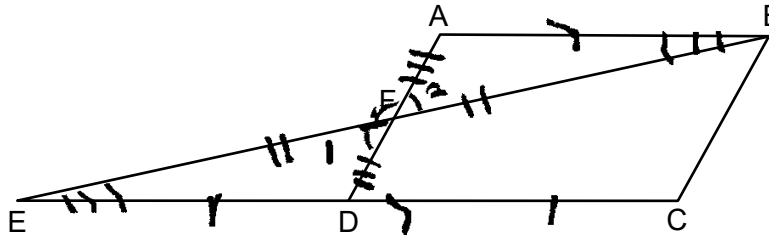

Prove *ABCD* is a parallelogram.

Statement	Reasons
1. ^{Quad}ABCD AFD and BFE bisect eachother $\overline{DE} \cong \overline{DC}$	1. Givens
2. $\overline{EF} \cong \overline{FB}$ and $\overline{AF} \cong \overline{DF}$	2. Bisectors split a seg into 2 ≅ seg.
3. ∠AFB ≅ ∠EFD	3. Vertical ∠'s are ≅
4. △ABF ≅ △DEF	4. SAS
5. $\overline{AB} \cong \overline{ED}$	5 CPCTC
6. $\overline{AB} \cong \overline{DC}$	6. Substitution
7. ∠ABF ≅ ∠FED	7. CPCTC
8. ABCD is a parallelogram	8. When 2 segments are equal length, and are ∥ then it makes it a parallelogram

Score 4: The student had a missing statement and reason to prove step 8. The student had an incomplete reason in step 8.

35 In quadrilateral *ABCD* below, side $\overline{CD}$ is extended through *D* to point *E* such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.
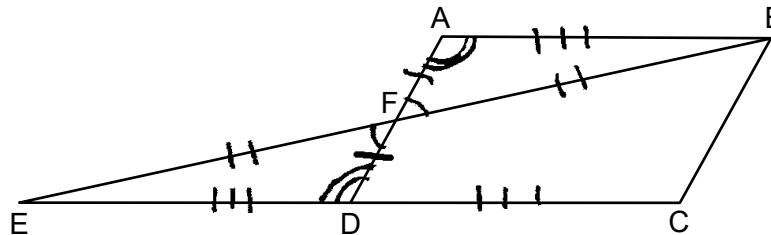

Prove *ABCD* is a parallelogram.

S	R
① $\overline{AFD}$ + $\overline{BFE}$ bisect eachother $\overline{DE} \cong \overline{DC}$, Quad ABCD	① Given
② ∡1 ≅ ∡2	② Vertical ∡'s are ≅
③ $\overline{EF} \cong \overline{BF}$ $\overline{AF} \cong \overline{DF}$	③ Bisect means to split into 2 ≅ parts
④ △FED ≅ △FBA	④ SAS
⑤ ∡E ≅ ∡ABF	⑤ CPCTC
⑥ $\overline{AB}$ // $\overline{EDC}$	⑥ Alt int ∡that are ≅ exist
⑦ ABCD //ogram	⑦ Opposite sides // opposite ∡ ≅ → parallelogram

Score 3: The student had two missing statements and reasons to prove step 7. The student had incorrect reasons in steps 6 and 7.

35 In quadrilateral *ABCD* below, side $\overline{CD}$ is extended through *D* to point *E* such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.

Prove *ABCD* is a parallelogram.   - SAS
                                      - Supplementary

statement

① $\overline{CD}$ is extended to point E, such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, + quad. ABCD $\overline{DE} \cong \overline{DC}$

② $\overline{EF} \cong \overline{FB}$ and $\overline{AF} \cong \overline{FD}$

③ $\angle EFD \cong \angle AFB$

④ $\triangle EFD \cong \triangle BFA$

⑤ $\angle A \cong \angle FDE$

⑥ $\angle FDC$ is supplementary to $\angle A$

⑦ ABCD is a parallelogram

reason

① given

② To bisect means to divide into 2 $\cong$ parts

③ If 2 lines intersect, vertical $\angle$s are $\cong$

④ SAS $\cong$ SAS

⑤ CPCTC

⑥ If angles are $\cong$ then a supplement to one of those $\angle$s will be supplementary to the $\cong$ angle

⑦ If a quad. has ~~supplementary~~ consecutive angles that are supplementary then it is a parallelogram.

Score 3:   The student had one missing statement and reason to prove step 6. The student made a conceptual error in step 7.

35 In quadrilateral *ABCD* below, side $\overline{CD}$ is extended through *D* to point *E* such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.
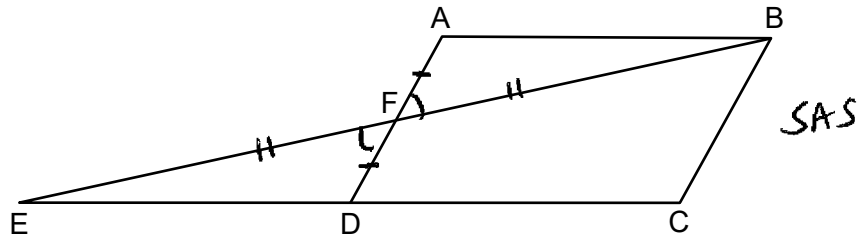
A   B

F

E   D   C
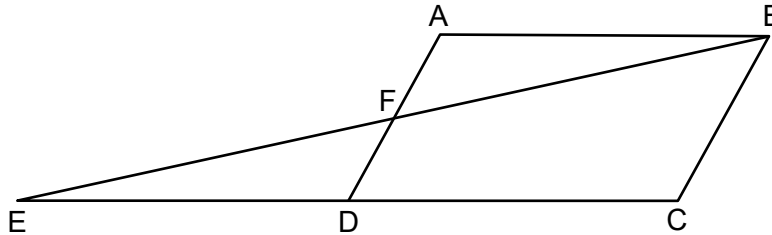
Prove *ABCD* is a parallelogram.

Statements	Reasons
Quad ABCD	
1) AFD and BFE bisect each other. DE≅DC	1) Given
2) AF≅DF, EF≅BF	2) Definition of a bisector.
3) ∠EFD ≅ ∠BFA	3) Vertical angles are ≅
4) △EFD ≅ △BFA	4) SAS
5) AB≅ED≅DC	5) Transitive Property
6) ∠A+∠D=180°	6) same side interior angles theorem.
∠D+∠C=180°	
7) AB//DC, AD//BC	7) Definition of parallel lines.
8) ABCD is a parallelogram	8) Definition of a parallelogram

Score 3:   The student proved △*EFD* ≅ △*BFA*. The student had no correct work after step 4.

35 In quadrilateral *ABCD* below, side $\overline{CD}$ is extended through *D* to point *E* such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.
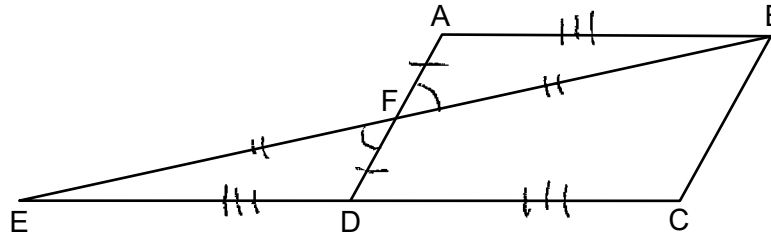

Prove *ABCD* is a parallelogram.

Statements	Reasons
1) QUAD ABCD, side $\overline{CD}$ is extended to pt E, $\overline{AFD}$ + $\overline{BFE}$ bisect each other, $\overline{DE} \cong \overline{DC}$	1) Given
2) $\overline{AF} \cong \overline{DF}$, $\overline{EF} \cong \overline{BF}$	2) def bisector
3) ∢DFE ≅ ∢AFB	3) verticle ∢'s
4) △EFD ≅ △BFA	4) SAS

SAS

Score 2:   The student had an incorrect reason in step 3.

35 In quadrilateral *ABCD* below, side $\overline{CD}$ is extended through *D* to point *E* such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.

Prove *ABCD* is a parallelogram.

1. Quad ABCD, $\overline{AFD}$ + $\overline{BFE}$ bisect each other, $\overline{DE} \cong \overline{DC}$	1. Given
2. ∢ AFB ≅ ∢ DFE	2. Vertical angles are ≅.
3. $\overline{AF} \cong \overline{FD}$	3. A bisector divides a segment into 2 ≅ parts.
4. $\overline{BF} \cong \overline{EF}$ △ DEF ≅ △ ABF	4. SSS ≅ SSS
5. $\overline{AB} \cong \overline{DC}$	5. opposite sides are ≅.
6. $\overline{AB} \parallel \overline{DC}$	6. opposite sides are ∥.
7. ABCD is a ▱	7. Proved in 4 + 5.

Score 2: The student had two correct statements and reasons in steps 2 and 3.

35 In quadrilateral *ABCD* below, side $\overline{CD}$ is extended through *D* to point *E* such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.
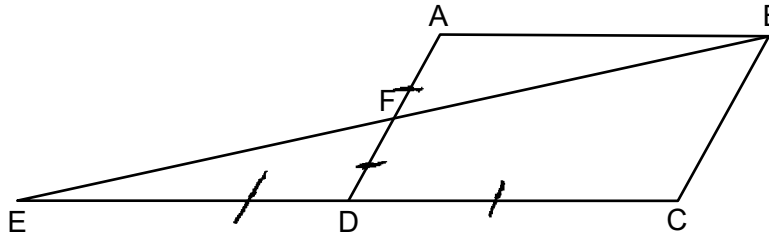
Prove *ABCD* is a parallelogram.

S	R
1. side $\overline{CD}$ is extended to point E / $\overline{AFD}$ & $\overline{BFE}$ bisect each other / $\overline{DE} \cong \overline{DC}$	1. Given
2. $\overline{AF} \cong \overline{FD}$	2. Segment that bisects a segment, creates 2 $\cong$ segments
3. $\overline{AD} \cong \overline{BC}$ $\quad$ $\overline{DC} \cong \overline{AB}$	3. Parallel lines are congruent
4. $\angle ABC \cong \angle ADC$ $\quad$ $\angle BCD \cong \angle DAB$	4. Angles made by parallel lines that are alternate interior angles are congruent.
5. ABCD is a parallelogram	5. 2 Pairs of congruent sides & angles.

Score 1: The student correctly proved $\overline{AF} \cong \overline{FD}$.

35 In quadrilateral *ABCD* below, side $\overline{CD}$ is extended through *D* to point *E* such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.
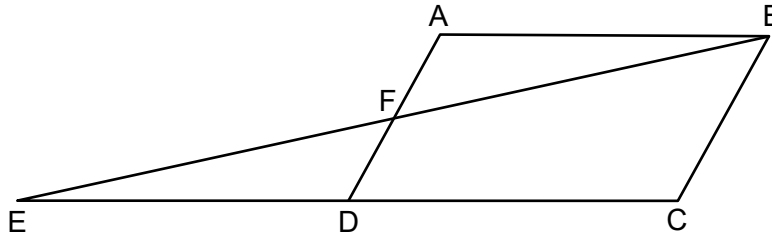

Prove *ABCD* is a parallelogram.

1. ABCD, $\overline{CD}$ is extended to point E, $\overline{AFD}$ and $\overline{BFE}$ bisect eachother, $\overline{DE} \cong \overline{DC}$	1. given
2. F is a midpt of $\overline{AD}$ ~~~~~~~~~	2. a seg bis. intersect a seg at its midpt
3. $\overline{AF} \cong \overline{DF}$ ~~~~~	3. a midpt ÷ a seg into 2 ≅ segs
4. $\overline{AB} \cong \overline{DC}$ $\overline{AD} \cong \overline{BC}$	4. opp sides in a ▱ are congruent
5. ABCD is a ▱	5. a ▱ has at least 1 pair of ≅ sides.

Score 1: The student correctly proved $\overline{AF} \cong \overline{FD}$.

35 In quadrilateral *ABCD* below, side $\overline{CD}$ is extended through *D* to point *E* such that $\overline{AFD}$ and $\overline{BFE}$ bisect each other, and $\overline{DE} \cong \overline{DC}$.

Prove *ABCD* is a parallelogram.

Statements	Reasons
1. Quad ABCD, $\overline{AFD}$ & $\overline{BFE}$ bisect each other, $\overline{DE} \cong \overline{DC}$	1. Given
2. $\overline{AD} \cong \overline{BC}$, $\overline{AB} \cong \overline{DC}$	2. Opposite sides of a quad are $\cong$.
3. ABCD is a parallelogram	3. When both pairs of sides are $\cong$ it's a parallelogram.

Score 0: The student did not show enough correct relevant work to receive any credit.